Premier examen partiel A 2013

QUESTION 1 (10 points total) Vantage points and line at infinity.

Givent straight lines $l_1 = (0, 2, 2)^T$, $l_2 = (0, 2, 5)^T$ and $l_3 = (-2, 0, 0)^T$ in the X-Y plane.

A) (1 point)
At which point $\tilde{x}_a$ in homogeneous coordinates does line l_1 intersect the line at infinity $l_\infty = (0, 0, 1)$?

B) (1 point)
At which point $\tilde{x}_b$ in homogeneous coordinates does line l_2 intersect the line at infinity $l_\infty = (0, 0, 1)$?

C) (1 point)
At which point $\tilde{x}_c$ in homogeneous coordinates does line l_3 intersect the line at infinity $l_\infty = (0, 0, 1)$?

D) (1 point)
At which point $\tilde{x}_d$ in homogeneous coordinates does line l_1 intersect the line at infinity l_3?

E) (1 point)
At which point $\tilde{x}_e$ in homogeneous coordinates does line l_2 intersect the line at infinity l_3?

F) (1 point)
At which point $\tilde{x}_f$ in homogeneous coordinates does line l_1 intersect the line at infinity l_2?

G) (1 point)
What are the real coordinates of (i.e. non-homogeneous) of $\tilde{x}_a$, $\tilde{x}_b$, $\tilde{x}_c$, $\tilde{x}_d$, $\tilde{x}_e$ and $\tilde{x}_f$?

H) (3 points)
What do you conclude on lines l_1, l_2 and l_3? Justify your answer by showing the three lines and points $\tilde{x}_a$, $\tilde{x}_b$, $\tilde{x}_c$, $\tilde{x}_d$, $\tilde{x}_e$ and $\tilde{x}_f$ in the same coordinate frame.

QUESTION 2 (20 points total) Frame transformations.

Let us assume the X-Y-Z Cartesian coordinate frame shown in Figure 1.

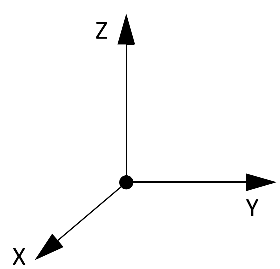

Figure 1 Frame coordinate for 2.

A) (4 points)
What is the matrix (in homogeneous coordinates) describing a $15°$ rotation around axis z?

B) (4 points)
What is the expression of the unit quaternion describing the same $15°$ rotation around axis z?

C) (4 points)

What is the expression of the unit quaternion describing a $15°$ rotation around axis X?

D) (8 points)

Given point $\underline{x}$ such that $\underline{x} = (7, 12, 25)^T$ in real (i.e. non-homogeneous) coordinates. What are the coordinates of $\underline{x}$ following a $15°$ rotation around axis Z followed by a $15°$ rotation around X of the same reference frame? Use the composition of rotations exploiting unit quaternions found in B and C for finding the solution.

QUESTION 3 (20 points total) Inverse perspective projection.

Given the non-inverting pinhole with focal length $F = 0.2m$ shown in Figure 2.

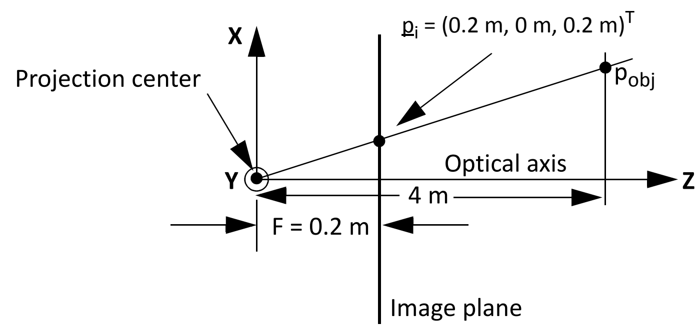

Figure 2 Non-inverting pinhole of Question 3

Image point $\underline{p_i}$ with coordinates $(0.2m, 0m, 0.2m)^T$ is observed in the reference frame of the pinhole (Y axis coming out of the page).

A) (5 points)

What is the parametric equation (i.e. $d = k + \lambda t$ form) of the projector passing through the center of projection and point $\underline{p_i}$?

B) (5 points)

What is the value of λ for the center of projection?

C) (5 points)

What is the value of λ for image point $\underline{p_i}$?

D) (5 points)

What is the value of λ for object point $\underline{p}_{obj}$ located at a distance of $4m$ from the center of projection along axis Z?

QUESTION 4 (25 points) Perspective projection using several reference frames.

Let us assume the geometry of the non-inverting pinholes in Figure 3.

Camera 1 is a non-inverting pinhole with focal length $F = 0.1m$. This camera, for which coordinate frame $X_1 - Y_1 - Z_1$ is initially superimposed on global reference frame $X_W - Y_W - Z_W$, is translated by $1m$ along X_W.

Camera 2 is a non-inverting pinhole with focal length $F = 0.1m$. This camera, for which coordinate frame $X_2 - Y_2 - Z_2$ is initially superimposed on global reference frame $X_W - Y_W - Z_W$, is translated by $2m$ along X_W.

Camera 3 is a non-inverting pinhole with focal length $F = 0.1m$. This camera, for which coordinate frame $X_3 - Y_3 - Z_3$ is initially superimposed on global frame $X_W - Y_W - Z_W$, is translated by $4m$ along Z_W and then rotated by $90°$ around axis Y of the frame after the translation (see Figure 3).

"Object" point P_W is observed by the three pinholes. The image coordinates of P_W on camera 1 and camera 2 are $p_{i1} = [0.01m\ 0m\ 0.1m]^T$ and $p_{i2} = [-0.01m\ 0m\ 0.1m]^T$ respectively.

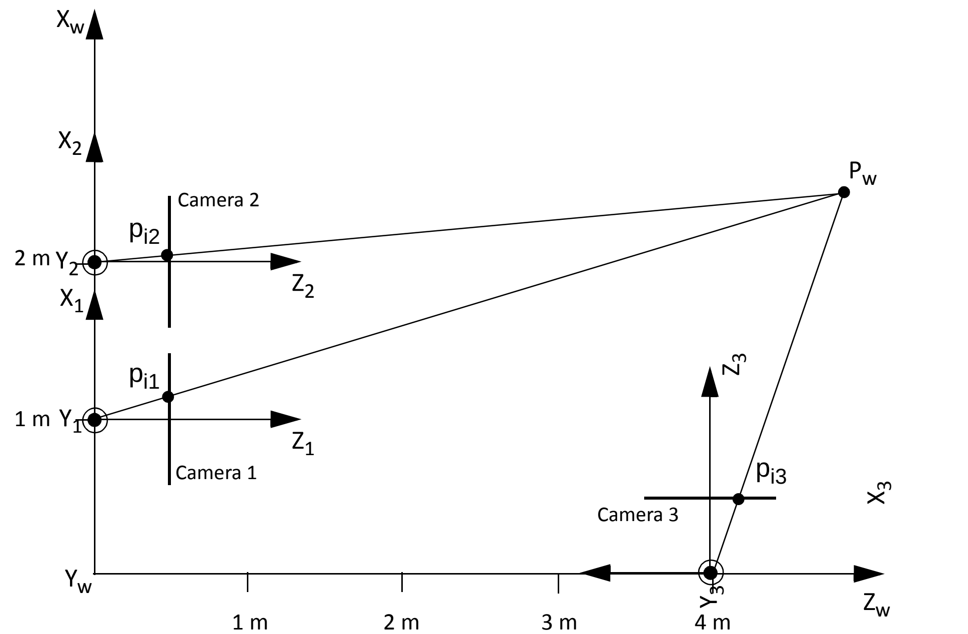

Figure 3 Geometry of the pinholes described in Question 4

What are the coordinates of image point p_{i3} of object point P_W on camera 3? Explain your line of reasoning for reaching the solution.

QUESTION 5 (25 points) Perspective projection and cross-ratio.

Let us assume the geometry in Figure 4 showing four object points A, B, C and D located on a straight line lying in the X-Y plane. The four points are observed by a non-inverting pinhole (Y axis coming out of the page) with center of projection "O" and lead to image points A', B', C' and D' on the image plane of the non-inverting pinhole. This question shows that when points aligned on a line lying in a plane are involved in a perspective projection, some distance measures are preserved despite the projection.

The *cross-ratio* between four points (A,B,C,D) on a straight line is defined as follows:

$$(A, B, C, D) = ((CA)/(CB))/((DA)/(DB)) \quad (1)$$

Using geometric principles exploiting similar triangles, show that cross-ratio (A,B,C,D) equals cross-ratio (A',B',C',D') under perspective projection.

Hint: draw a line parallel to O-A passing through B and intersectiong line O-C in N and line O-D in. Draw a line parallel to O-A passing through B' and intersecting line O-C in N' and line O-D in M'. Explain your line of reasoning.

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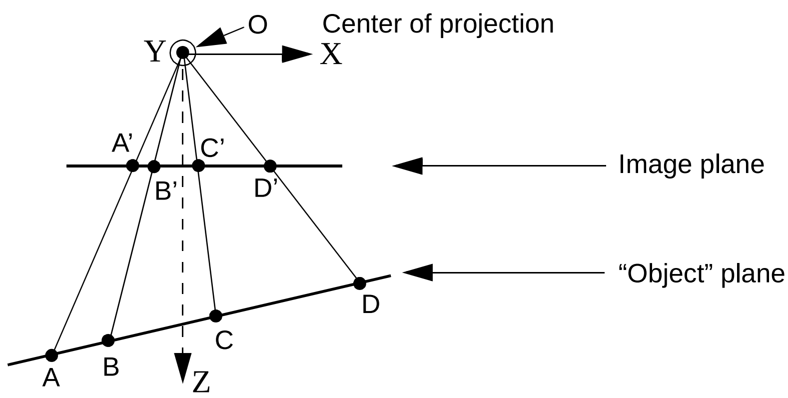

Figure 4 Geometry for Question 5.