Département de génie électrique
Faculté des sciences et de génie

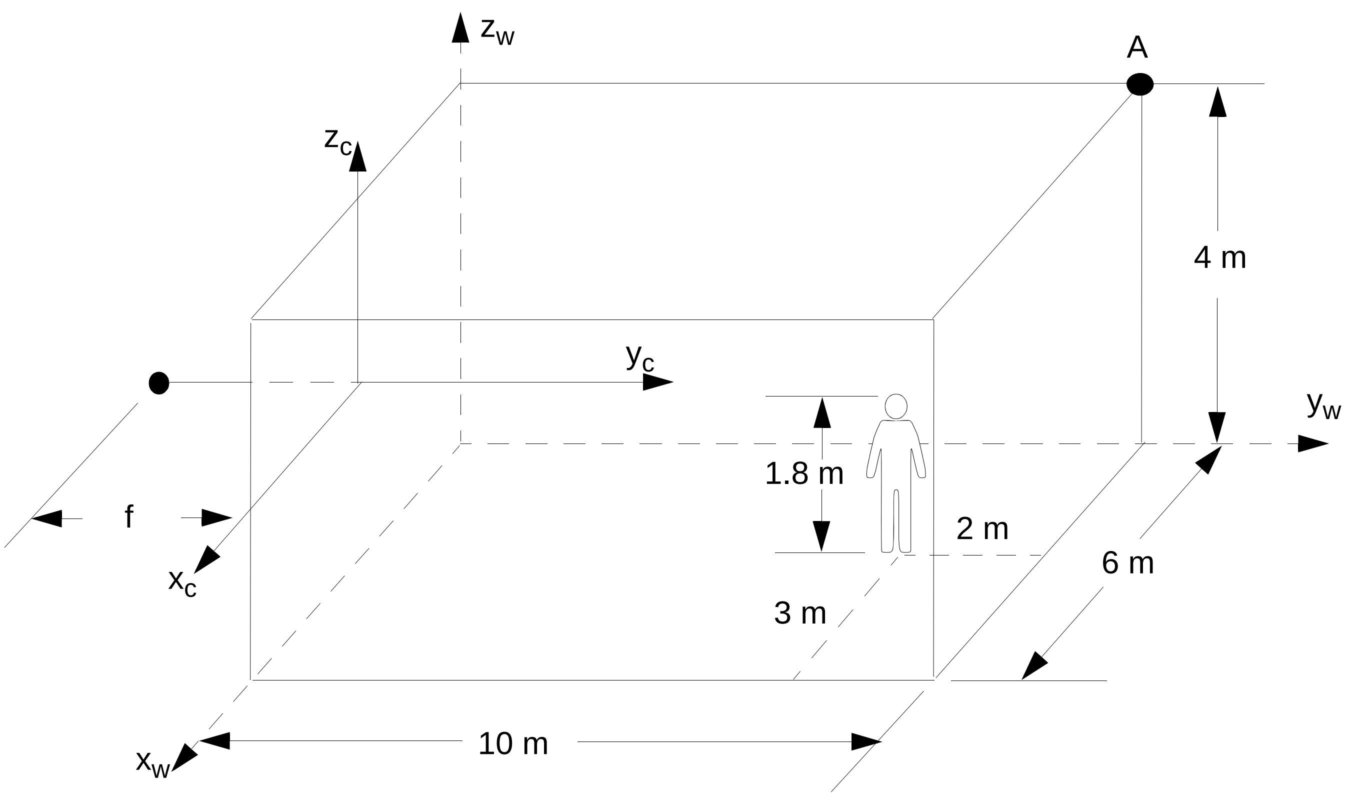
Vision numérique
GIF-4100 / GIF-7001

Mid-term exam A 2010

QUESTION 1 (20 points) Perspective projection.

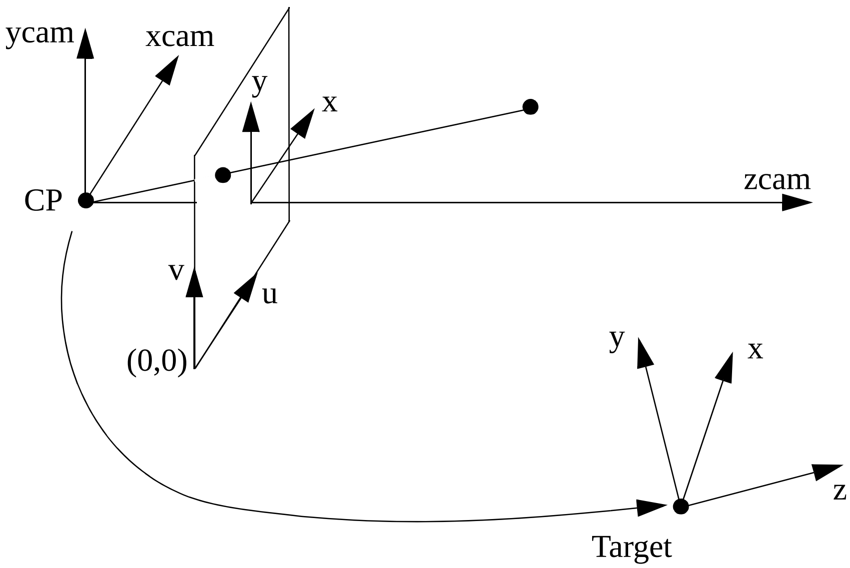
Suppose that a surveillance camera based on the non-inverting pinhole model with focal length f = 0.05, and with axes xc, yc, zc (yc being the optical axis), is placed in the middle of a wall with axes xw, yz, zw,. The room has the dimensions shown in the figure below. Find the image coordinates of point A in the room and the image coordinates of the top of the head of the person shown in the figure.

Figure 1 Geometry for QUESTION 1



QUESTION 2 (20 points) Camera matrix

Consider a camera which one models with a non-inverting pinhole whose constant f = 10 mm, pixel size is 10 x 10 microns (1 micron = 10-3 mm), the photosensitive matrix is 640 columns x 480 lines pixels and the optical axis passes through the image plane at (u, v) = (320, 240) pixels, which is located exactly at the center of the image. The lines and columns of the photosensitive matrix are perfectly perpendicular. Please carefully note the otientation of the frame reference in the following diagram.



A) (5 points) Write the 3x3 matrix of the intrinsic parameters.

B) (5 points) Predict the image coordinates (u,v) (in pixels) of the projection of point (-75,65,410) mm known in the camera reference frame. Use the matrix calculated in A).

C) (5 points) For an image point at (u,v) = (200,300) pixels, write the equation of the projector in the form of two planes and calculate the intersection of this projector with plane z -700m = 0.

D) (5 points) When the camera was calibrated, **R** and **t** where such that $\mathbf{X}_{cam} = \mathbf{R}\,\mathbf{X}_{target} + \mathbf{t}$ with:

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & -\frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix}, \boldsymbol{t} = \begin{bmatrix} 10 \\ 40 \\ 1000 \end{bmatrix}$$

Find the 3 unit vectors which describe the orientation of each of the 3 axes (OX, OY and OZ) of the camera reference frame in the target reference frame.

QUESTION 3 (points) Homography

A) (5 points) How many points (minimum), must be placed in correspondence to estimate the 9 parameters of a homography (induced by a plane) between the image plane of a camera and the image plane of another camera? Explain.

B) (5 points) Write the equation (or equations) provided by each point ((u1,v1) <=> (u2,v2)) in correspondence.

C) (5 points) Which geometric model describes each of the equations obtained in B) (is it a point, a line, a plane, a circle, etc ...)?

D) (5 points) One has a projector such as the one available in the classroom, as well as a camera and a computer. With a laser pointer, one projects a point on the wall and moves this point. One would like the projector to also project a point which will continually follow and superpimpose on the laser point on the wall. Explain how an homography can enable the calibration and exploitation of this system.

QUESTION 4 (20 points) Vanishing points

Consider a line in three-dimensional space defined in the following manner:

$$\underline{x} = \underline{a} + \lambda \underline{b} \tag{1}$$

where $\underline{a}$ is a point in space:

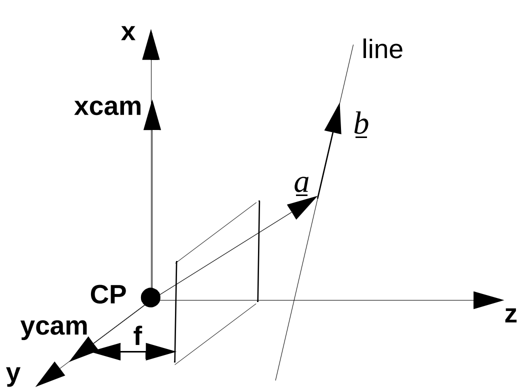
$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2}$$

and $\underline{b}$ is a unit vector whose components are the direction cosines of the direction of the line in space (the direction cosines are the cosines of the angles made by a unit vector with the axes of a cartesian coordinate frame, i.e. they are the components of the unit vector along each axis of the cartesian reference frame. The following constraint is verified: $(\cos(\theta_x))^2 + (\cos(\theta_y))^2 + (\cos(\theta_z))^2 = 1$):

$$\underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{3}$$

Figure 2 shows this line.

Figure 2 Geometry for QUESTION 4



A point on this line thus has the following coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 + \lambda b_1 \\ a_2 + \lambda b_2 \\ a_3 + \lambda b_3 \end{bmatrix} \qquad (4)$$

A) (7 points) What are the coordinates x_p and y_p of the perspective projection of a point of this line on the image plane of a non-inverting pinhole with focal length f and the center of projection located at the origin of the pinhole with optical axis z.

B) (7 points) What will become of x_p and y_p when $\lambda \to \infty$?

C) (6 points) Let $x_{p,\lambda \to \infty}$ et $y_{p,\lambda \to \infty}$ be the value of x_p and y_p obtained in B). This pair of coordinates is called the vanishing point of the line. If an one has acquired an image of the line and measured the coordinates of the vanishing point, can one calculate the components of vector $\underline{b}$? If so, what are these components?

QUESTION 5 (20 points) Radiometry

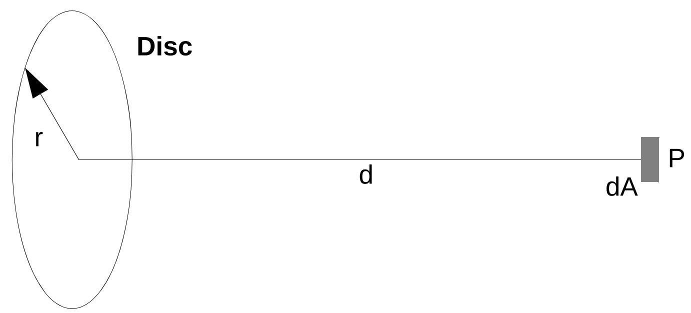
Consider a luminous disc of radius r whose luminance L is uniform in all directions of the hemisphere on all points of the disc (i.e. the disc is Lambertian).

A) (5 points) Provide the illuminance received by a surface element dA located at a distance d on the disc axis (see Figure 3). Consider that the disc radius is not negligible with respect to the distance d.

The following integration formula could be useful:

$$\int \frac{x\,dx}{(c^2 + x^2)^2} = \frac{-1}{2(c^2 + x^2)} \; . \qquad (5)$$

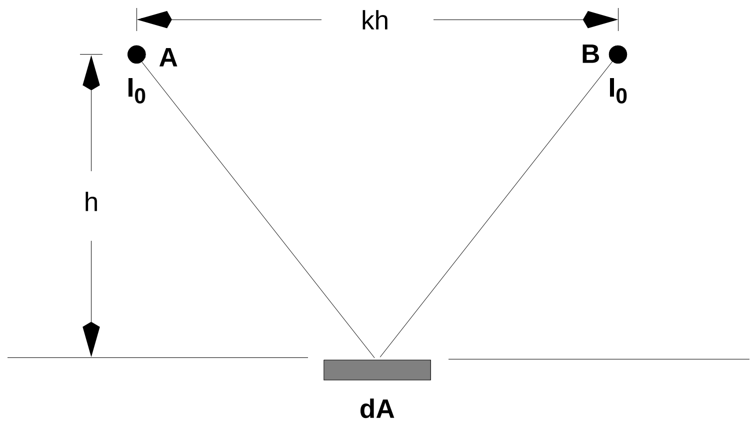
Figure 3 Lambertian disc in QUESTION 5



B) (5 points) Consider the geometry in Figure 4 showing two uniform point sources A and B with intensity

I_0 and located at height h meters from the ground and kh meters apart.

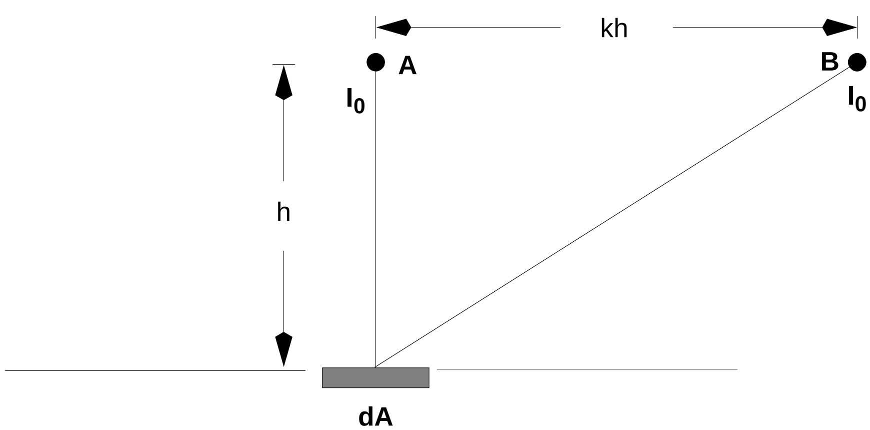
Figure 4 Geometry for the problem in QUESTION 5 B)



Calculate the illuminance E received by a small surface element of area dA placed on the ground exactly midway between A and B.

C) (5 points) Now calculate the illuminance if the surface element dA is located directly under point source A as shown in Figure 5.

Figure 5 Geometry for the problem in QUESTION 5 C).



D) (5 points) Compute the ratio ρ between the illuminance computed at QUESTION 5 B) and the illuminance computed at QUESTION 5 C) for $k = 1$ and $k = 2$.