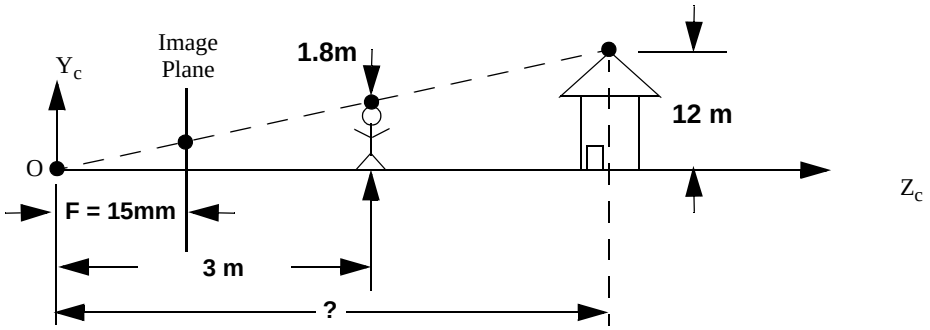
Département de génie électrique
Faculté des sciences et de génie

Mid-term exam A 2011

QUESTION 1 (15 total points) Perspective projection in a camera-centered coordinate system.

A) (10 points) Consider the non-inverting pinhole camera of focal length F = 15 mm in Figure 1. A person with a height of 1.80 m is placed at a distance of 3 m from the pinhole camera according to the z axis which corresponds to the optical axis. At what distance should one place a building with a height of 12 m so that the image will be the same as that of a point located exactly at the top of the head of the person?

Figure 1 Geometry of Question 1



B) (5 points) What will the distance be if the focal length of the pinhole camera doubles and now has a length of 30 mm?

QUESTION 2 (30 total points) Perspective projection and camera matrix.

Consider the diagram in Figure 2 which shows a pinhole camera of focal length F = 10 mm and axes y_C and z_C (optical axis) observing a scene in the global Cartesian coordinate system X_G, Y_G, Z_G with X_G entering the page and perpendicular to the page. Axis x_C of the camera is a vector which also enters the page and is perpendicular to the page. The camera was moved a distance of 1 m along the y axis and 1 m along the z axis in the global coordinate system before being rotated by $-45°$ with respect to the X axis of the coordinate system having been subjected to this first translation.

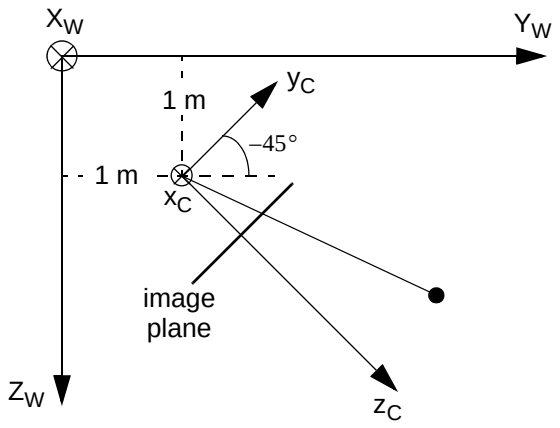
The photosensitive sensor of the pinhole camera has a resolution of 640 columns by 480 lines of pixels. A pixel has a dimension of $10\ \mu m$ x $10\ \mu m$ ($1\ \mu m = 10^{-6}\ m$). The optical axis of the pinhole camera intersects the image plane exactly in the center at the pixel with the coordinates (320, 240).

A) (12 points) What is the matrix of the intrinsic parameters?

B) (12 points) What is the matrix of the extrinsic parameters?

C) (6 points) What is the camera matrix which describes the complete geometric process of the formation of the image by perspective projection?

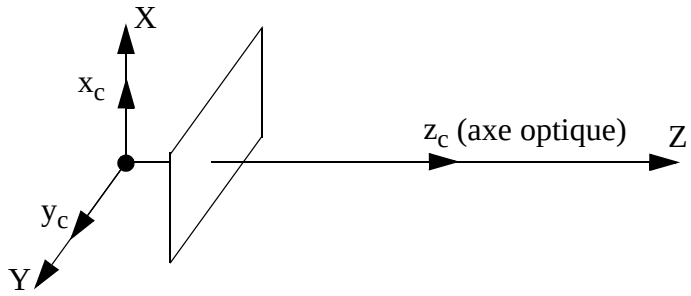
Figure 2 Geometry of Question 2



QUESTION 3 (20 total points) Perspective projection.

Consider the non-inverting pinhole camera of axes xc-yc-zc and focal length 1 installed in the global coordinate system X-Y-Z as shown in Figure 3.

Figure 3 Geometry of Question 3.



Let there be a straight line l_1 in space (coordinate system X-Y-Z) with equation:

$$\vec{l_1} = \vec{a_1} + \lambda \vec{d_1} \tag{1}$$

and a second straight line $\vec{l_2}$ in space (coordinate system X-Y-Z) with equation:

$$\vec{l_2} = \vec{a_2} + \beta \vec{d_2}\,. \tag{2}$$

A) (5 points) Give the expression of the coordinates of image points (u_{l1}, v_{l1}) and (u_{l2}, v_{l2}) corresponding to the points on line 1 and on line 2.

B) (5 points) What happens to the coordinates of the image points when $\lambda \to \infty$ and $\beta \to \infty$?

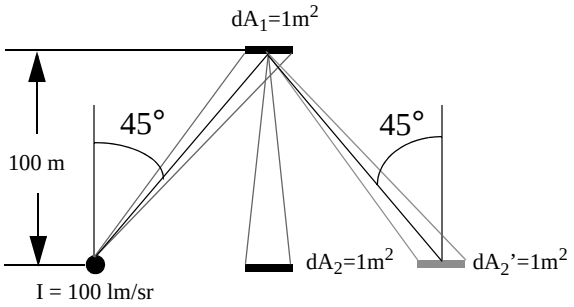
C) (5 points) What relationship must the components $\vec{d}_1$ and $\vec{d}_2$ respect so that the coordinates of the points found in Question 3 B (above) are equal?

D) (5 points) Give a geometric interpretation of the answer found in Question 3 C above.

QUESTION 4 (18 total points) Radiometry

Consider the schematic of Figure 4 which shows a point source of intensity $I = 40000$ lm/sr illuminating a Lambertian surface element dA_1 with an area of 1m^2 located at a height of 100 m and making a 45° angle with respect to the vertical of the position of the source. The relfectivity (albedo) of the surface dA_1 est $\rho = \frac{1}{\sqrt{2}}$.

Figure 4 Schematic of Question 4.



A) (10 points) What is the illuminance dE_2 received by the surface element dA_2 of area 1m^2 ?

B) (8 points) What is the illuminance dE_2' received by the surface element dA_2' of area 1m^2 whose normal is making a 45° angle with the vertical of the position of the surface element dA_1 as shown in the figure?

QUESTION 5 (17 total points) Homography

Consider a pinhole camera which moves from observation position 1 to observation position n as shown in Figure 5. The camera's field of view is shown in the figure. At each position the camera acquires an image of the facade of a building, located at a distance, where the variations in the facade's thickness are negligible with respect to the distance at which the camera is located (i.e. $d \gg 1$ and $h \ll d$). Suppose now that an overlap in the field of view exists between each pair of consecutive images in the sequence (i.e. 1 and 2 are overlapping, 2 and 3 are overlapping, etc.). This means that for each pair of images, a part of the building is observed in both images.

Explain how homography can be used to reconstruct the mosaic image of the building in the coordinate system of camera 1. Discuss the advantages of this approach and its limitations.

Figure 5 Geometry of Question 5.