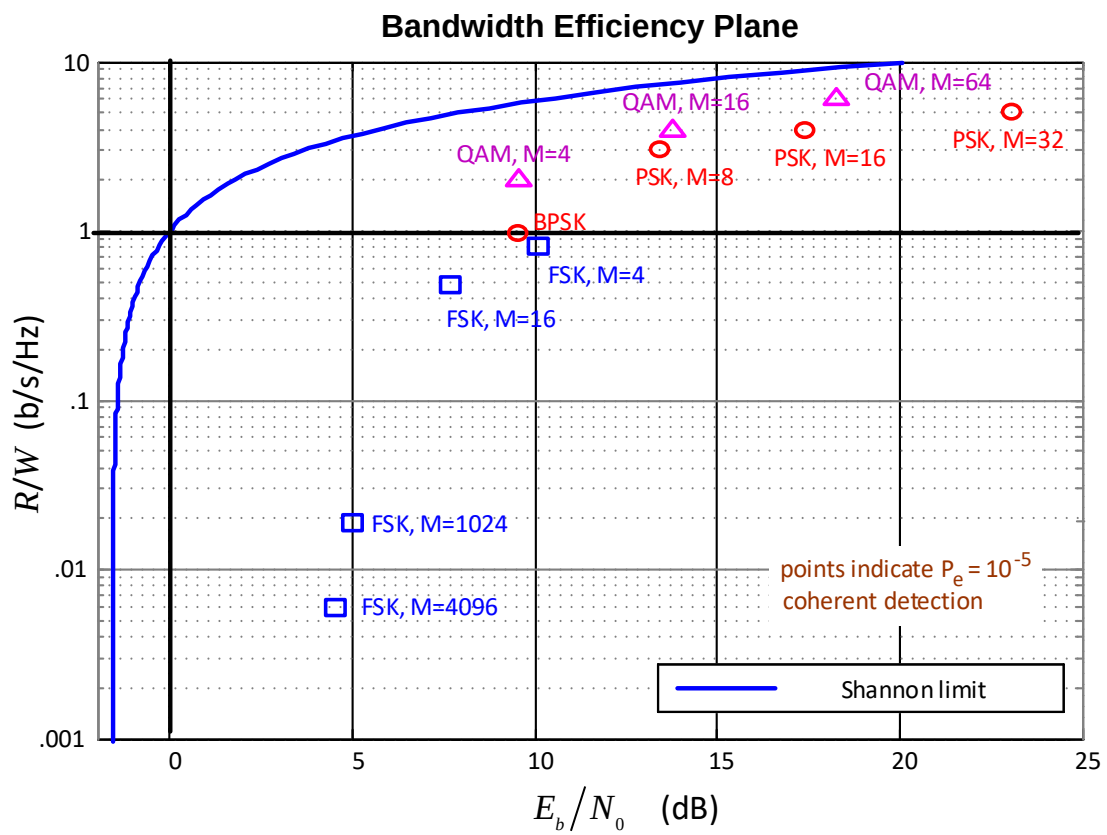
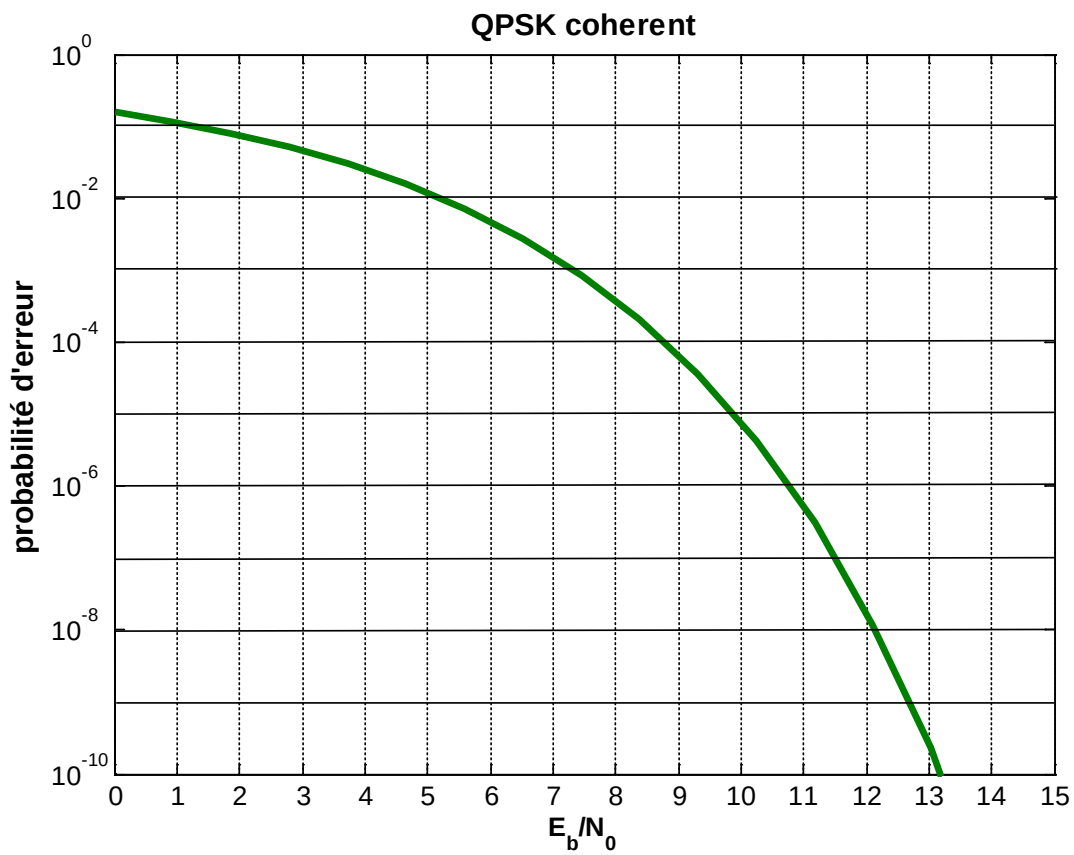
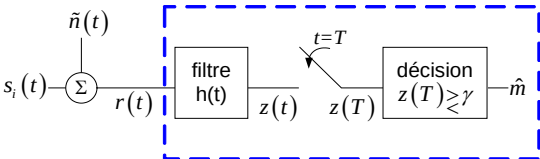
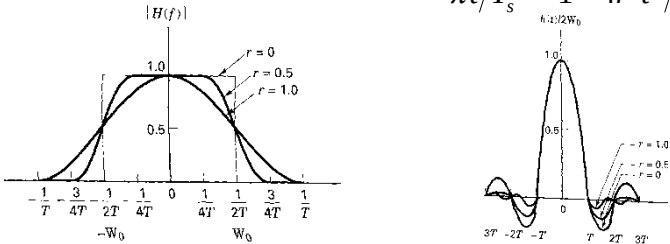
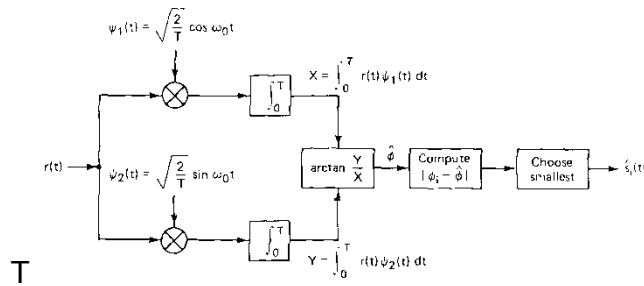




Sampling receiver 	MAP: i that maximizes $p(z s_i) p(s_i)$ i that minimizes $\ \mathbf{r} - \mathbf{s}_i\ ^2 - N_0 \ln P(\mathbf{s}_i)$ $P(\mathbf{s}_i)$ = a priori probability of symbol $\mathbf{s}_i$ ML: i that maximizes $p(z s_i)$ i that minimizes $\ \mathbf{r} - \mathbf{s}_i\ ^2$
Raised cosine pulse $v(t) = \frac{\sin(\pi t/T_s) \cos(r\pi t/T_s)}{\pi t/T_s \frac{1 - 4r^2 t^2/T_s^2}{T_s^2}}$ 	Average energy per symbol $E_s = \frac{1}{M} \sum_{i=1}^M \ \mathbf{s}_i\ ^2$ $= \frac{1}{M} \sum_{i=1}^M [\text{energy of signal } i]$ Energy per bit vs. energy per symbol $E_b \log_2 M = E_s$
Conversion of I/Q space to signal space $(\tilde{a}_n^I, \tilde{a}_n^Q) = \sqrt{\frac{M \cdot E_s}{\sum_{i=1}^M [(a_n^I)^2 + (a_n^Q)^2]}} (a_n^I, a_n^Q)$ Coordinates Signal Space (blue arrow pointing to $\tilde{a}_n^I, \tilde{a}_n^Q$) Coordinates I/Q Space (red arrow pointing to a_n^I, a_n^Q)	Union bound $P_e \approx \frac{2K}{M} Q\left(\frac{D_{\min}}{\sqrt{2N_0}}\right) = \frac{2K}{M} Q\left(d_{\min} \sqrt{\frac{E_b}{N_0}}\right)$ K is the number of pairs of signals separated by the minimum distance $D_{\min}$
QAM $\eta = \log_2 M$ square QAM case $M=L^2$ $P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3 \log_2 M}{(M-1)} \frac{E_b}{N_0}}\right) \quad d_{\min} = \sqrt{\frac{6 \log_2 L}{L^2 - 1}}$	Minimum distance in signal space $D_{\min} = \min_{i \neq k} \ \mathbf{s}_i - \mathbf{s}_k\ \quad \text{and} \quad d_{\min} = \frac{D_{\min}}{\sqrt{2E_b}}$
$P_e(\text{BPSK}) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ $P_e(\text{OOK}) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$ $P_e(\text{QPSK}) \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ Loss with respect to QPSK $d_{\min} = \sqrt{x} \sqrt{2} \quad \text{perte} = -10 \log_{10} x$	For orthogonal modulation $P_e(\text{bit}) = P_b = P_e(\text{symbol}) \frac{M/2}{M-1}$ For non-orthogonal modulation with gray coding $P_e(\text{bit}) = P_b = \frac{P_e(\text{symbol})}{\log_2 M}$ Spectral efficiency $\eta = \frac{R_b}{W} = \frac{1}{T_b} \frac{1}{W} \text{ bits/s}$

MPSK coherent detection

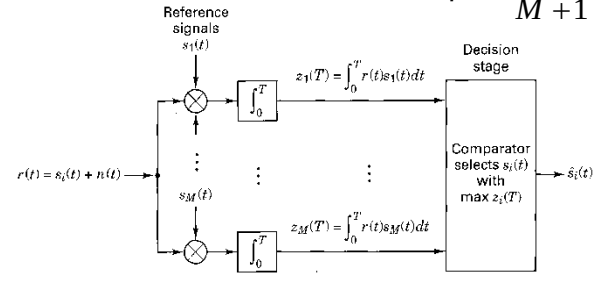
$$\eta = \log_2 M^\dagger$$



$$P_e(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{M}\right) = 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin \frac{\pi}{M}\right)$$

MFSK coherent detection

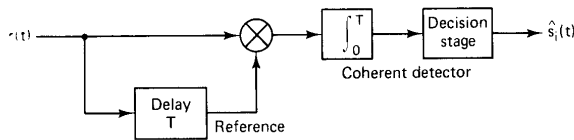
$$\eta = \frac{2 \log_2 M}{M+1}^\dagger$$



$$P_e = (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = (M-1)Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

Minimum separation $1/2T_s$

DPSK non-coherent detection



Loss of 1 dB between DPSK and BPSK $P_e = \frac{1}{2} e^{-E_b/N_0}$

Trigonometric relationships

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\cos 2\theta = 2\cos^2 \theta - 1 = 1 - \sin^2 \theta \quad \tan \theta = \sin \theta / \cos \theta$$

Gram Schmidt Process

$$\psi_1(t) = \frac{1}{\sqrt{E_1}} s_1(t) \quad \text{Where } E_1 \triangleq \int_0^T s_1^2(t) dt$$

$$\theta_2(t) \triangleq s_2(t) - \langle s_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$E_2 \triangleq \int_0^T \theta_2^2(t) dt \quad \psi_2(t) = \frac{\theta_2(t)}{\sqrt{E_2}}$$

$$1. \quad \theta_i(t) = s_i(t) - \sum_{k=1}^{i-1} \langle s_i(t), \psi_k(t) \rangle \psi_k(t)$$

$$E_i \triangleq \int_0^T \theta_i^2(t) dt \quad \psi_i(t) = \frac{\theta_i(t)}{\sqrt{E_i}}$$

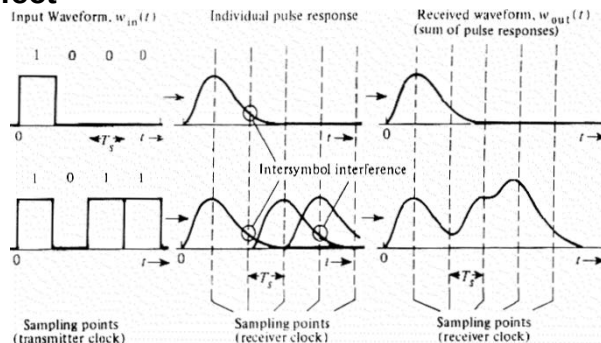
Shannon's Law

$$C = W \log_2(1 + SNR)$$

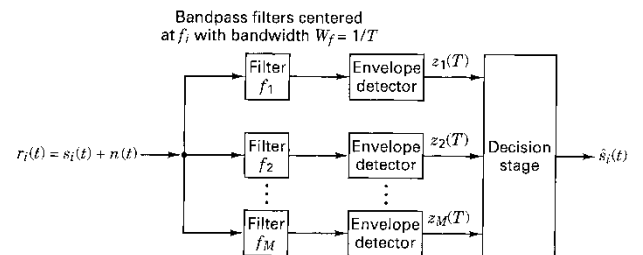
$$SNR = \frac{E_b R_b}{N_0 W}$$

$$\frac{E_b}{N_0} = \frac{W}{C} (2^{C/W} - 1) \quad \frac{C}{W} \rightarrow 0 \Rightarrow \frac{E_b}{N_0} \rightarrow -1.6 \text{ dB}$$

ISI effect



MFSK noncoherent detection $\eta = \frac{\log_2 M}{M}^\dagger$



$$P_e(BFSK) = \frac{1}{2} e^{-E_b/2N_0}$$

Loss of 1 dB between coherent and non-coherent detection for BFSK

Minimum Separation $1/T_s$

[†] In assuming an ideal Nyquist pulse