

The Multitone Channel

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Abstract—Multitone quadrature amplitude modulation (QAM) is an interesting candidate for data transmission over linear channels with frequency dependent transfer functions. In this paper, the maximum bit rate of multitone QAM over a general linear channel is found. First, the overall bit rate for an AWGN channel with a two-level transfer function is maximized, using a multitone QAM system. The power distribution between the tones and the number of bits/symbol per tone is optimized for a given symbol error rate. Extending these results to the general channel, it is shown that the optimum power division for multitone signals is similar to the water-pouring solution of information theory. Furthermore, multitone QAM performance is about 9 dB worse than the channel capacity, independent of the channel characteristics. The multitone results throughout are compared to those of an equivalent single-tone linearly equalized system. The comparison shows that the multitone system is useful for some channels, e.g., those with deep nulls in the transfer function. The maximum bit rate over a twisted-pair channel which is performance dominated by near-end crosstalk (NEXT) is also found.

I. INTRODUCTION

THERE is, at present, interest [1] in the concept of multitone transmission, i.e., the use of several parallel QAM subchannels to transmit data over channels with linear distortion. One of the main questions concerning this technique is how to maximize the bit rate through the channel under given requirements, e.g., fixed transmitter power and equal probabilities of error on all subchannels. This paper describes the maximization of the overall bit rate R_b through the optimization of the power distribution, and the number of signaling levels, for each tone (subchannel). In the limit, we consider an infinite multitone system transmitting over a general linear channel.

For analytical convenience, we first approach the problem of a channel with a transfer characteristic consisting of two bands, each of constant (but different) attenuation, with one QAM tone occupying the bandwidth of each band. The results of the two-level channel are compared to those of a single-tone transmission over the entire channel bandwidth using linear equalization. The comparison shows that the optimized two-tone system may show significant improvement as compared to the single-tone channels, e.g., when there is a deep null in the channel frequency characteristic. The two-level results are then generalized to the problem of a channel with a continuous transfer function $H(f)$. Interestingly enough, it is found that the solution of the optimum power distribution resembles the water-pouring solution of information theory. We also have considered the multitone performance of a twisted-pair channel which is performance dominated by near-end crosstalk (NEXT).

Section II discusses the general channel model. Section III contains the optimization of the two-tone channel and Section

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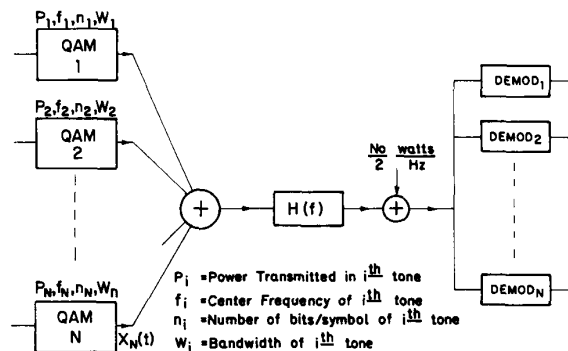


Fig. 1. The multitone QAM system.

IV describes the optimization for the continuous transfer function channel. In Section V, the multitone performance over a twisted-pair channel which is NEXT-dominated is investigated. Finally, the conclusions are discussed in Section VI.

II. THE QAM MULTITONE SYSTEM

The multitone QAM system under consideration is shown in Fig. 1. We first assume that $H(f)$ is a staircase frequency transfer characteristic as shown in Fig. 2. In the limit as $W_i \rightarrow 0$, $H(f)$ approaches a continuous function of frequency and we have an "infinite" multitone QAM signal. The transmitted signal consists of N QAM signal tones (each with a rectangular Nyquist spectrum of bandwidth equal of W_i Hz).

Each transmitted signal tone is given by

$$x_i(t) = \sum_{n=-\infty}^{\infty} a_n s(t - nT_i) \cos 2\pi f_i t + \sum_{n=-\infty}^{\infty} b_n s(t - nT_i) \sin 2\pi f_i t \quad (1)$$

where $T_i = 1/W_i$, and f_i is the carrier frequency of the i^{th} signal. The baseband signal $s_i(t)$ is a minimum bandwidth Nyquist signal free of intersymbol interference (ISI).

The symbols a_n and b_n take on values of $\pm 1, \pm 3, \dots$ depending on the number of signal points in the symmetric QAM signal (e.g., for 16 QAM, a_n and $b_n = \pm 1, \pm 3$) and $s(t)$ is normalized so that the average energy per symbol equals E . The individual QAM signals have rectangular (or cross) signal constellations of the type described in [2]. The minimum distance between points is equal to $2a$.

The transmitted power of each QAM signal tone is equal to P_i and the total transmitted power is equal to P watts, i.e.,

$$P = \sum_{i=1}^N P_i. \quad (2)$$

Each M -ary QAM tone may have a different number of bits per symbol where $M = 2^{n_i}$, n_i being the number of bits per

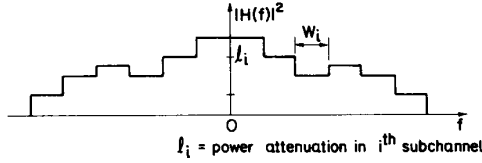


Fig. 2. Piecewise-constant transfer function.

symbol, for the i th signal. The overall bit rate R_b , is given by

$$R_b = \sum_{i=1}^N n_i W_i \text{ bits/s.} \quad (3)$$

The receiver consists of bandpass filters of width W_i followed by coherent demodulators and samplers at the instant of maximum signal voltage.

We are interested in maximizing R_b (through optimal power division amongst the tones and optimal choice of bits per symbol for each tone) under the restrictions that the total transmitted power P be limited, and that the symbol error probabilities for all tones be equal. In the next section, we will maximize R_b for a two-tone signal with equal bandwidths, $W_1 = W_2 = W$, (i.e., two levels of attenuation) with total transmitted power and symbol error rate constraints. These results will then be extended to channels with general continuous frequency characteristics.

III. OPTIMIZATION OF OVERALL BIT RATE FOR TWO TONES

For the two-tone bandwidth case $|H(f)|^2$ is as shown in Fig. 2, with only the first two attenuation levels being nonzero. The level $l_1 = 1$ and $l_2 = l$, and the overall bit rate R_b , is given by

$$R_b = (n_1 + n_2) W. \quad (4)$$

The total transmitted power P , is given by

$$P = P_1 + P_2. \quad (5)$$

For even values of n , the probability of symbol error for QAM signaling, for the case of high signal-to-noise (see [3, ch. IV]) with constellation of size $M = 2^{n_i}$ signals is given by

$$\Pr \{ \epsilon \} = 4 \left(1 - \frac{1}{\sqrt{2^{n_i}}} \right) Q \left[a \sqrt{\frac{2}{N_0}} \right] \quad (6)$$

where the Q function is defined by [3] as

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy. \quad (7)$$

The value a is related to the average symbol energy E for n even by the equation below [2]

$$a^2 = \frac{3E}{2} / (2^n - 1). \quad (8)$$

For odd values of n , the probability of error and the average energy/symbol can be closely approximated by (6) and (8). We also use these expressions for noninteger values of n , although a noninteger value of n is usually impractical, such systems will be considered in our systems for analytic convenience.

We now maximize the total bit rate R_b of this two-tone system, under the conditions that $P/N_0 W$ is given, and that the probabilities of symbol error for each subchannel must be equal to $\Pr \{ \epsilon \}$. The power division between the two tones, and the number of bits/symbol per tone are optimized to find the maximum bit rate, $R_{b_{\max}}$.

The symbol error probabilities for each subchannel are given by

$$\Pr_1 \{ \epsilon \} = K_{n_1} Q \left[a_1 \sqrt{\frac{2}{N_0}} \right] \quad (9)$$

and

$$\Pr_2 \{ \epsilon \} = K_{n_2} Q \left[a_2 \sqrt{\frac{2}{N_0}} \right]$$

where the constants K_{n_i} are functions of n_i , and assume values in the interval

$$2 \leq K_i < 4. \quad (10)$$

We define

$$P_1 = kP \text{ and } P_2 = (1-k)P \quad (11)$$

where k is the share of total power P in tone one. Substituting for " a " [using (8)], and after some manipulation, we have

$$n_1 = \log_2 \left[1 + \frac{3kP/N_0 W}{\left[Q^{-1} \left(\frac{\Pr \{ \epsilon \}}{K_{n_1}} \right) \right]^2} \right] \quad (12)$$

and

$$n_2 = \log_2 \left[1 + \frac{3(1-k)P/N_0 W}{\left[Q^{-1} \left(\frac{\Pr \{ \epsilon \}}{K_{n_2}} \right) \right]^2} \right]$$

where $Q^{-1} \{ \cdot \}$ is the inverse Q function. Using (4) and (12), and maximizing R_b with respect to k we find that the optimum value of k is

$$k_{\text{opt}} = \frac{1}{2} \left[1 + \frac{1}{lM_2} - \frac{1}{M_1} \right] \quad (13)$$

and that the optimum values of n_1 and n_2 are

$$n_{1\text{opt}} = \log_2 [1 + k_{\text{opt}} M_1] \quad \text{and} \quad n_{2\text{opt}} = \log_2 [1 + (1 - k_{\text{opt}}) l M_2] \quad (14)$$

where

$$M_1 = \frac{3P/N_0 W}{\left[Q^{-1} \left\{ \frac{\Pr \{ \epsilon \}}{K_{n_1}} \right\} \right]^2} \text{ and } M_2 = \frac{3P/N_0 W}{\left[Q^{-1} \left\{ \frac{\Pr \{ \epsilon \}}{K_{n_2}} \right\} \right]^2}. \quad (15)$$

By choosing $K_{n_1} = K_{n_2} = 4$, $R_{b_{\max}}$ is lower bounded, and by choosing $K_{n_1} = K_{n_2} = 2$, $R_{b_{\max}}$ is upper bounded.

For both the upper and lower bounds,

$$k = \frac{1}{2} \left[1 + \frac{1}{M_0} \left(\frac{1}{l} - 1 \right) \right], \quad n_1 = \log_2 \frac{1}{2} \left[1 + M_0 + \frac{1}{l} \right] \text{ and } n_2 = n_1 + \log_2 l \quad (16)$$

(where $M_0 = M_1 = M_2$).

In Fig. 3, we have plotted k_{opt} and the upper and lower bounds (R_{ub}/W and R_{lb}/W), respectively, as a function of $P/N_0 W$ for $l = 1/16$ (-12 dB). The probability of error is 10^{-5} .

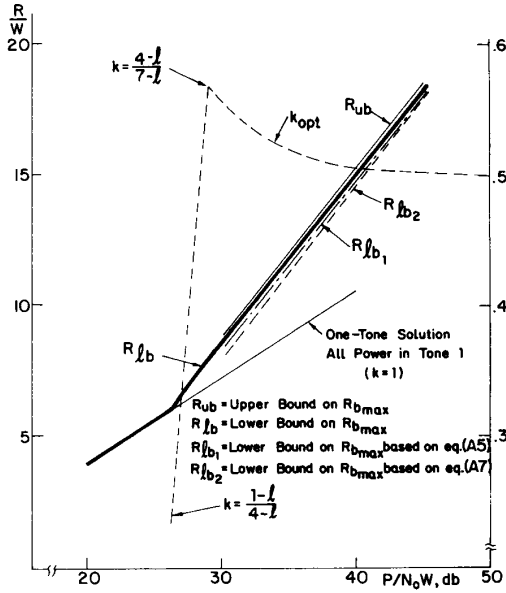


Fig. 3. Bounds on maximum bit rate $R_{b\max}$ for a two-level channel, two-tone solution, and one-tone solution as a function of P/N_0W , dB ($l = -12$ dB). k_{opt} is also shown.

As we see from the figure the bounds are tight. For values of M smaller than $4/l - 1$ [corresponding to $k = (1 - l)/(4 - l)$], the optimum power division is simply to put all the transmitter power in the unattenuated tone. For a small region of P/N_0W corresponding to $(1 - l)/(4 - l) < k < (4 - l)/(7 - l)$, the lower bound is based on the fact that in this region the optimum value of $n_2 = 2$, and this accounts for the sharp incline in the k_{opt} curve. We have also found two other simpler but looser lower bounds on $R_{b\max}$. These bounds, R_{lb1} and R_{lb2} , are described in the Appendix and are also shown in Fig. 3.

A. Comparison to One-Tone Modulation

The optimum results found in this section are now compared to those classical one-tone methods, using linear equalization, for improving channel performance. Channels with linear equalization at the receiver only, and channels with equalization at both transmitter and receiver, are considered.

In this case of a linear equalizer at the receiver, the overall transfer characteristic (from input to output) generates an output QAM signal with no intersymbol (ISI) in a frequency band of $2W$ Hz. This system is shown in Fig. 4 with $H_T(f) = 1$.

For a perfectly flat channel of bandwidth $2W$, the $\text{Pr}\{\epsilon\}$ (for $n > 4$) is given approximately by [5]

$$\text{Pr}\{\epsilon\} \approx \left[\exp -\frac{3}{4} \frac{P}{N_0 W 2^n} \right] \quad (17)$$

where n = number of bits per symbol.

In the case of a two-level channel with a linear equalizer at the receiver only, the noise term is enhanced [6] by a factor of $[2/(1 + 1/l)]^{-1}$. After some manipulation, we find the bit rate R_{b1} for this equalized channel, is given by

$$R_{b1} = n(2W) = 2W \log_2 \frac{3}{4} \left[\frac{2}{1 + \frac{1}{l}} \right] \frac{P/N_0W}{(-\text{Ln Pr}\{\epsilon\})} \text{ bits/s.} \quad (18)$$

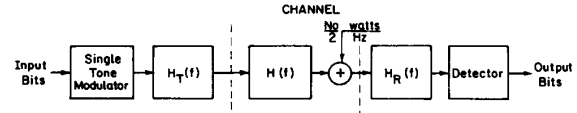


Fig. 4. Single-tone linear equalized channel.

For the channel, with half the equalization at the transmitter and half at the receiver, i.e., $H_T(f) = H_R(f) = 1/(\sqrt{H(f)})$ [6], the signal-to-noise ratio is enhanced by a factor of $[4/(1 + (1/\lambda)^{1/2})^2]^{-1}$ (see [6]), and the bit rate R_{b2} for this channel is given by

$$R_{b2} = 2W \log_2 \frac{3}{4} \left(\frac{2}{1 + 1/l^{1/2}} \right)^2 \frac{P/N_0W}{(-\text{Ln Pr}\{\epsilon\})} \quad (19)$$

Therefore, we see that (A.6), (18), and (19) may be respectively rewritten as

$$R_{lb2}/W = M_l + \log_2 l$$

$$R_{b1}/W = M_l + 2 \log_2 [2/(1 + 1/l)] \quad (20)$$

and

$$R_{b2}/W = M_l + 2 \log_2 [4/(1 + 1/l^{1/2})^2]$$

where M_l is defined as

$$M_l = 2 \log_2 \left[\frac{3}{4} \frac{P/N_0W}{(-\text{Ln Pr}\{\epsilon\})} \right] \quad (21)$$

Normalized values of $R_{b\max}$ (where $R_{b\max}$ is the optimum two-tone solution R_{lb2}), R_{b1} and R_{b2} are plotted in Fig. 5 as a function of l .

As we see, the optimal two-tone solution $R_{b\max}$, substantially outperforms the linear equalization techniques for small values of l , i.e., channels with deep nulls.

IV. OPTIMIZATION OF MULTITONE QAM FOR A GENERAL CHANNEL, $|H(f)|^2$

We now turn our attention to a general linear channel with channel characteristic $|H(f)|^2$. Tight upper and lower bounds on (infinite) multitone QAM transmission, through this channel, will be found. The results indicate the possibilities and bounds of a finite multitone system.

The channel transfer function $|H(f)|^2$ is first approximated by a staircase function, as shown in Fig. 6. The transmitted power P , is divided into N tones, each tone, with power $k_i P$ watts, and carrying n_i bits per symbol where

$$\sum_{i=0}^{N-1} k_i = 1. \quad (22)$$

Transmission is allowed only in subchannels for which $n_i > 2$. This assumes that QAM requires at least one bit in each dimension. By taking the summation over all subchannels for which $n_i > 2$ we find that the total bit rate R_b , is equal to

$$R_b = \sum_{i=0}^{N-1} n_i \cdot \Delta f = \sum_{k=0}^{N-1} \Delta f \cdot \log_2 \left[1 + \frac{(3k_i P/N_0 \Delta f) |H_i(f)|^2}{\left[Q^{-1} \left\{ \frac{\text{Pr}\{\epsilon\}}{K} \right\} \right]^2} \right] \quad (23)$$

We use K instead of K_n , so that our results as mentioned

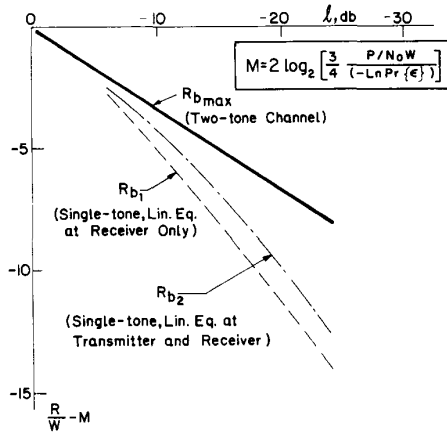


Fig. 5. Normalized maximum bit rate $R/W - M$ as a function of the channel attenuation for two-tone and linear equalized single-tone channels.

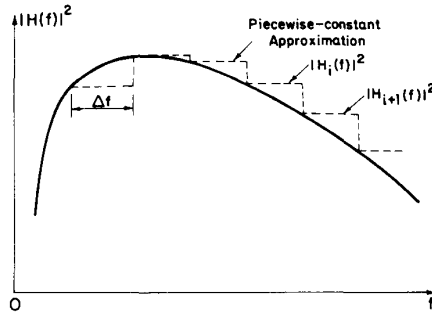


Fig. 6. General channel transfer function, $|H(f)|^2$ (and piecewise-constant approximation).

before are actually the upper and lower bounds on R_b depending on whether K equals two or four.

As Δf becomes smaller the staircase approximation of the channel transfer function approaches $|H(f)|^2$. In this case, we have an infinite multitone system and as $\Delta f \rightarrow 0$ the bit rate R_b is given by

$$R_b = \int_{f \in F_A} df \log_2 \left[1 + \frac{3}{[Q^{-1}\{\epsilon\}]^2} \frac{P}{N_0} k(f) |H(f)|^2 \right] \quad (24)$$

where

$$\int_{f \in F_A} k(f) df = 1, \quad k(f) > 0.$$

F_A is the frequency range in which the integrand is greater than two.

In order to maximize R_b , the normalized power distribution $k(f)$ is optimized. This problem is very similar to the information theory problem of maximization of the capacity of a linear channel [7], and the solution has the same form, i.e., the "water-pouring" solution. The optimum normalized power distribution $k_0(f)$ is found to be

$$k_0(f) = \begin{cases} \lambda - \frac{1}{K_0 |H(f)|^2} & \text{for } f \in F_A \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

where

$$\int_{f \in F_A} k_0(f) df = 1, \quad K_0 = \frac{P}{N_0} \frac{3}{[Q^{-1}\{\epsilon\}]^2}$$

and λ is a Lagrange multiplier.

The maximum bit rate, $R_{b_{\max}}$ is given by

$$R_{b_{\max}} = \int_{f \in F_A} df \log_2 \{ \lambda K_0 |H(f)|^2 \}. \quad (26)$$

From the condition that the integrand must be greater than two, we have that $f \in F_A$ if

$$|H(f)|^2 > \frac{4}{K_0 \lambda}. \quad (27)$$

An iterative process is required, to jointly solve (25) and (26), to find $k_0(f)$ and λ .

The water-pouring solution for optimizing channel capacity is a very satisfying result because it agrees with the philosophical argument of Shannon for achieving capacity, as well as similar arguments made in [8] in reference to PCM. In fact, if we relax the restriction of (24), to $\log \{\cdot\} > 0$, we find that for $\text{Pr}\{\epsilon\} = 10^{-5}$ and $K = 4$, there is about an 8.4 dB degradation in performance when comparing "infinite" multitone to the capacity for any $|H(f)|^2$, independent of its shape.

For large signal-to-noise ratios, the results above reduce to

$$R_{b_{\max}} \cong \int_{f \in F_A} df \log_2 \{ K_0 |H(f)|^2 \} \quad (28)$$

where

$$K_0 = \frac{3}{2} \frac{P}{N_0 W_{\text{eff}}} / (-\text{Ln} [\text{Pr}\{\epsilon\}])$$

and W_{eff} is the measure of the positive frequency range of F_A as defined in (24) (i.e., $\log_2 \{\cdot\} > 2$).

As a simple example of the use of the above equations, we reconsider the two-level channel of Section II for the high signal-to-noise case (and $\text{Pr}\{\epsilon\} = 10^{-5}$). Using (28), we find

$$\frac{R_b}{W} = \log_2 \left[\frac{9}{16} \frac{1}{[-\text{Ln} \text{Pr}\{\epsilon\}]^2} \left(\frac{P}{N_0 W} \right)^2 l \right] \quad (29)$$

(similar to the results of Section II).

If $P/N_0 W$ equals 30 dB and l equals 1/16, we find $R_b/W = 8.05$. If we compare this to the same result for a linear equalizer (at the receiver) for the same channel [see (18)], we find $R_b/W = 5.87$, i.e., the multitone solution represents a possible performance improvement of over 35 percent.

As another example (with an analytic solution), we consider a channel with a Gaussian transfer function given by the equation below

$$|H(f)|^2 = e^{-(|f| - f_c)^2 / B^2} \quad \text{where } f_c \gg B. \quad (30)$$

In this case, we assume $P/N_0 B$ equals 30 dB and $\text{Pr}\{\epsilon\}$ equals 10^{-5} . Using (25) and (26), after some manipulations, we find

$$\frac{R_{b_{\max}}}{B} = \frac{W_{\text{eff}}}{B} \frac{1}{\text{Ln } 2} \cdot \left\{ \text{Ln} \left[\frac{3}{2} \frac{P/N_0 B}{(-\text{Ln} \text{Pr}\{\epsilon\})} \frac{1}{W_{\text{eff}}} \right] - \frac{1}{12} \left(\frac{W_{\text{eff}}}{B} \right)^2 \right\}. \quad (31)$$

W_{eff} is the bandwidth of the range F_A . For the values of $P/$

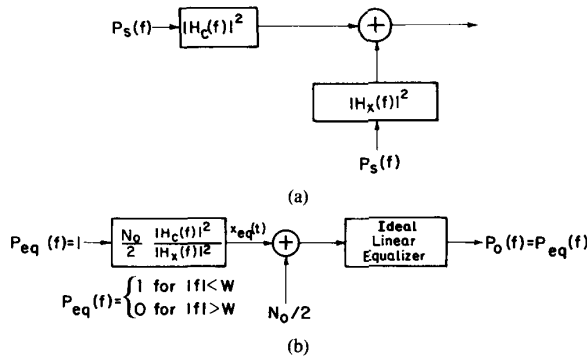


Fig. 7. (a) Twisted-pair channel model. (b) Equivalent NEXT-dominated channel model (linear equalizer includes $2/N_0 |H_x(f)|^2 P_s(f)$ term from whitening filter).

$N_0 B$ and $\Pr\{\epsilon\}$ given above, W_{eff}/B equals 3.07 and $R_{b_{max}}/B$ equals 13.12.

These results are now compared to the maximum bit rate for a linear equalized (receiver only) single-tone QAM system as found in [5] for the same Gaussian channel. In [5], with the same constraints described above (for $P/N_0 B$ and $\Pr\{\epsilon\}$), optimum transmission bandwidth equals $2.76B$ and the maximum bit rate $R_{b_{max}}$ equals $12.0B$.

The bit rate of multitone QAM in this case is only about 8 percent higher than that of a single-tone QAM. Interestingly enough, optimum multitone QAM requires more bandwidth.

As in the two-level case, it is expected that multitone QAM may perform favorably as compared to single-tone linearly equalized systems for channels containing continuous channel characteristics with nulls or sharply decreasing amplitude characteristics, e.g., the twisted-pair channel, which will be described in the next section.

V. THE TWISTED-PAIR CHANNEL (NEXT-DOMINATED)

The twisted-pair cable is a very important element of local area networks (LAN) and integrated service data networks (ISDN). The cable consists of many twisted pair wires in very close proximity to each other. The dominating influence on the receiver detection performance for a given pair is the near-end crosstalk (NEXT) [9]–[11] generated by transmitters located at the same end of the cable as the receiver. The transmitted signals from other twisted pairs “feed” into the receiver, and this NEXT signal is the dominant noise factor when determining detectability performance. It is usually assumed [10], [11] that the nearest five to seven twisted pairs cause most of the crosstalk.

A model for the twisted pair channel including the NEXT effect has been proposed [9], [10] and is shown in Fig. 7(a). (The model ignores the inherent white Gaussian noise (WGN) of the channel because the NEXT dominates, but the WGN can easily be taken into account.)

The model includes the twisted pair wire attenuation function $|H_c(f)|^2$ which as a first-order approximation may be assumed [9], [10] to be

$$|H_c(f)|^2 = e^{-\alpha \sqrt{f}} \quad (32)$$

where

$$\alpha = k \frac{l}{l_0}.$$

The frequency f is measured in KHz, k is a physical constant of the twisted-pair, l_0 is a reference length (e.g., 18 000 ft), and l is the length of the twisted pair.

The NEXT effect is modeled [9] as a signal with the same spectral density as the transmitted signal (since all pairs are assumed to be transmitting similar signals), passing through a crosstalk transfer function, $|H_x(f)|^2$ where

$$|H_x(f)|^2 = \beta f^{3/2} \quad (33)$$

β is a constant of the cable.

The interesting fact about this channel is that both the transmitted signal and the interfering NEXT signals are of the same spectral density.

The NEXT-dominated channel model used in the analysis of twisted pair cables also has a maximum multitone bit rate which may be found using the results of the last section.

The equivalent white-noise model of the NEXT-channel is shown in Fig. 7(b) where $P_{eq}(f)$ is an effective input signal, which without loss of generality may be chosen as having uniform power spectral density equal to one, over an effective bandwidth W_{eff} .

For the NEXT-dominated channel

$$P = 2 W_{eff}$$

and

$$k(f) = 1/W_{eff}. \quad (34)$$

Using the above in (26), we have

$$R_b = \int_0^{W_{eff}} df \log_2 \left[1 + M_n \frac{W_{eff}}{N_0} \frac{1}{W_{eff}} \left(\frac{N_0}{2} \right) \frac{|H_c(f)|^2}{|H_x(f)|^2} \right] \quad (35)$$

where

$$M_n = 3 / \left(\left[Q^{-1} \left\{ \frac{\Pr\{\epsilon\}}{K} \right\} \right]^2 \right).$$

$|H_c(f)|^2$ and $|H_x(f)|^2$ are, respectively, the linear channel and crosstalk, transfer functions of the NEXT-dominated model. Therefore,

$$R_b = \int_0^{W_{eff}} df \log_2 \left[1 + M_n \frac{|H_c(f)|^2}{|H_x(f)|^2} \right]. \quad (36)$$

where W_{eff} is the highest frequency for which

$$|H_c(f)|^2 > \frac{3}{M_n} |H_x(f)|^2. \quad (37)$$

Equations (36) and (37) may now be used to find the multitone capabilities of a NEXT-dominated channel.

As a numerical example we use the expressions for $|H_c(f)|^2$ and $|H_x(f)|^2$ of (32) and (33) with $\alpha = 1.158$, $l_0 = 18\,000$ ft, and $\beta = 10^{-9}$. Using these values in (36) for a 600 ft cable with $\Pr\{\epsilon\} = 10^{-6}$, we find that the maximum bit rate R_b is 49.7 Mbits/s (with $W_{eff} = 8.5$ MHz).

We can now compare these results to the optimum results [5] for a single-tone QAM system with linear equalization for the twisted pair channel [using the equivalent channel model of Fig. 7(b)]. In this case, we find that the maximum bit rate R_b for a single-tone system is 34.8 Mbits/s ($W_{eff} = 8.2$ MHz). Using multitone it may be possible to achieve an improvement of more than 40 percent as compared to single-tone QAM.

VI. CONCLUSIONS

The maximum bit rate which can be transmitted using multitone QAM through a linear additive white Gaussian noise channel with a continuous transfer-function $H(f)$ was found.

The optimum power distribution for a multitone QAM signal was found to be similar in form to the water-pouring solution of information theory. The performance of the multitone QAM system is about 9 dB worse than channel capacity independent of the channel transfer function.

The multitone results were also compared to those of a linear-equalized channel for a two-level channel and for a Gaussian channel. For the two-level channel with a deep null, we found that large improvement was possible using multitone while the improvement for the Gaussian channel was not as great. However, it is expected that multitone QAM may show greater improvement for channels with sharply decreasing amplitude characteristics, such as the twisted-pair channel. The expression for the maximum bit rate over a NEXT-dominated twisted pair channel has also been found in our work. Quantitative results for a specific 600 ft twisted-pair cable show that multitone QAM transmission outperforms single-tone QAM by more than 40 percent.

APPENDIX

A looser lower bound on $R_{b\max}$ can be found using the simple bound [4] on the Q function (for large values of x) given below

$$Q(x) \leq \frac{1}{2} e^{-(x^2/2)}. \quad (A.1)$$

Using the inequality above it can be shown that for large values of P/N_0W

$$k_{\text{opt}} = \frac{1}{2} + \frac{m}{2} \left[\frac{1}{l} - 1 \right] \quad (A.2)$$

where

$$m = -\frac{2 \text{Ln Pr} \{\epsilon\}}{3 P/N_0W}. \quad (A.3)$$

The optimal values of n'_1 and n'_2 are now given by

$$n'_1 = \log_2 \frac{1}{2} \left[1 + \frac{1}{m} + \frac{1}{l} \right] \quad (A.4)$$

and

$$n'_2 = n'_1 + \log_2 l \quad \text{for } n'_2 > 2.$$

In this case, the lower bound R_{lb_1} is given by

$$R_{lb_1} = (2n'_1 + \log_2 l) W. \quad (A.5)$$

R_{lb_1}/W is also shown in Fig. 3 (for $l = 1/16$) and as is seen it is tight, especially at high values of P/N_0W . For values of P/N_0W less than that for which $n_2 = 2$, we simply use the one tone solution to lower bound, $R_{b\max}$.

The bounding (approximation) process may be taken one more step, by using the bound (approximation for $n > 4$).

$$2^n - 1 \leq 2^n.$$

In this case, k_{opt} is simply equal to one-half, (as long as the number of bits/symbol is greater than two, for each tone), i.e., the power is evenly divided between both tones. The bits/

symbol, n'_1 and n'_2 are

$$n'_1 = \log_2 \frac{3}{4} \frac{P/N_0W}{(\text{Ln Pr} \{\epsilon\})} \quad (A.6)$$

and

$$n'_2 = n'_1 + \log_2 l, \quad \text{for } n'_2 > 2.$$

Therefore, the new lower bound R_{lb_2} is given by

$$R_{lb_1} > R_{lb_2} = (2n'_1 + \log_2 l) W.$$

R_{lb_2} is also shown in Fig. 3.

For values of P/N_0W less than that for which $n'_2 = 2$, we assume that the one-tone solution (in a manner similar to that described for R_{lb_1}).

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