

Equalization

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Equalization

- Sklar, Section 3.4, pp.149-161
- B. Sklar, "How I learned to love the trellis," in *IEEE Signal Processing Magazine*, vol. 20, no. 3, pp. 87-102, May 2003.
- R. E. Ziemer and W. H. Tranter, "Principles of Communications: Systems, Modulations, and Noise," 4th ed: John Wiley & Sons, 2014, pp. 233-237, 455-466.
- G. Baudoin, "Radiocommunications numériques: principes, modélisation et simulation," Paris: Dunod, chapitre sur l'égalisation.

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Maximum Likelihood Sequence Estimator MLSE

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Source of ISI $BW_{channel} = I \times BW_{signal}$

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Intersymbol interference

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ML receiver

- Without ISI
 - examine ONE symbol interval
 - Choose the nearest one

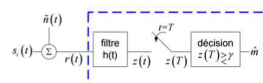
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ML receiver

➤ Without ISI

- examine ONE symbol interval
- Choose the nearest one



➤ With ISI

- examine a symbol **SEQUENCE**
- sequence as long as the **channel memory**

$$\{i(k)\}_{k=-N}^0 \text{ that minimizes } \sqrt{\|r(-NT) - s_{i(N)}\|^2 + \dots + \|r(-kT) - s_{i(k)}\|^2 + \dots + \|r(0) - s_{i(0)}\|^2}$$

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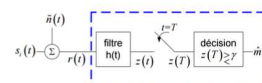
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ML receiver

➤ Without ISI

- examine ONE symbol interval
- Choose the nearest one



➤ With ISI

- examine a symbol **SEQUENCE**
- sequence as long as the channel memory

decision is the
entire sequence

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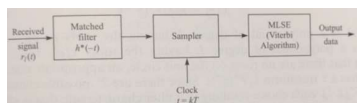
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MLSE

➤ Maximum Likelihood Sequence Estimator

- Optimal
 - Can eliminate ISI completely for some channels
- Extremely complex
 - for each bit examine the possible M^N sequences!

- Viterbi algorithm can help with complexity



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ML receiver with VA

➤ Comprehensive approach

- examine M^N SEQUENCES of symbols
- Choose the best - calculating M^N metrics each interval

$$\{i(k)\}_{k=-N}^0 \text{ qui minimise } \sqrt{\|r(-NT) - s_{i(N)}\|^2 + \dots + \|r(-kT) - s_{i(k)}\|^2 + \dots + \|r(0) - s_{i(0)}\|^2}$$

seule décision retenue

➤ Viterbi approach

- Use a trellis; trellis size determined by M^N

Recurrence to reduce complexity of operations - memory

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MLSE avec Viterbi

➤ Convolutional Codes

- TX encoder introduces a correlation between bits
- RX decoder uses the Viterbi algorithm to exploit the correlation to correct errors

➤ An ISI channel

- introduces correlations between bits
- ISI is deterministic, not random
- MLSE can use the Viterbi algorithm to exploit correlation for better detection

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AWGN vs ISI

➤ AWGN is random

- the matched filter optimizes the SNR
- no filter can "predict" or "equalize" this noise

➤ ISI is deterministic

- The matched filter does not touch these effects
- filtering can improve ISI (an equalizer)
- MLSE is the optimal solution
 - is not a filter
 - is non-linear

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Simulations and performance

MLSE

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MLSE

- Channel knowledge required
 - "encoder" if it were a FEC
- Complexity of the Viterbi algorithm
 - M points in the constellation
 - Channel memory of L
 - M^{L+1} states

Parameters

Specify channel via:

Channel coefficients: $[1 \ 0.7 \ 0.5 \ 0.3]$

Signal constellation: $[1+j \ -1+j \ -1-j \ 1-j]$

Traceback depth: 21

Function Block Parameters: MLSE Equalizer

MLSE Equalizer (msk) (bks)

Equalize a linearly modulated signal through a dispersive channel using the Viterbi algorithm.

The block estimates the transmitted sequence from the received signal and the channel estimates, and then outputs complex constellation points at the symbol rate.

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Butterworth Channel

- MLSE completely eliminates the ISI

The plot shows Gain in dB on the y-axis (from -60 to 10) versus Frequency on the x-axis (log scale from 0.01 to 100). Five curves are shown for different Butterworth filter orders: 1st order, 2nd order, 3rd order, 4th order, and 10th order. All curves start at 0 dB at low frequencies and roll off at higher frequencies. The 10th order curve has the steepest roll-off, reaching -60 dB at a frequency of approximately 1.5.

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Butterworth Channel

- MLSE completely eliminates the ISI

The plot shows BER on the y-axis (log scale from 10^{-6} to 10^0) versus Eb/No (dB) on the x-axis (linear scale from 0 to 16). A single curve for Ideal BPSK is shown, which starts at a BER of 10^{-1} at 0 dB and decreases to 10^{-6} at approximately 10 dB.

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Example

- Channel

$[0.227 \ 0.460 \ 0.688 \ 0.460 \ 0.227]$

- narrower than the signal
- large fades

The plot shows Normalized Magnitude Squared (dB) on the y-axis (from -40 to 10) versus Normalized Frequency (rad/s) on the x-axis (from -3 to 3). The curve is a sinc function centered at 0, with a main lobe reaching 0 dB and side lobes reaching approximately -20 dB at ± 2 rad/s.

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Example

- Channel

$[0.227 \ 0.460 \ 0.688 \ 0.460 \ 0.227]$

- Matlab
- Eqberdemo

The plot shows BER on the y-axis (log scale from 10^{-6} to 10^0) versus Eb/No (dB) on the x-axis (linear scale from 0 to 16). Five curves are shown: Ideal BPSK (blue line), Linear Equalizer (red line with '+' markers), DFE (green line with '+' markers), Ideal MLSE (magenta line with '+' markers), and Imperfect MLSE (black line with '+' markers). The Ideal BPSK curve is the lowest, followed by the Linear Equalizer, DFE, Ideal MLSE, and Imperfect MLSE.

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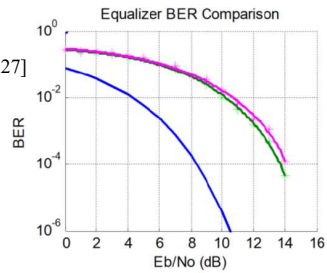
Example

➤ Channel

[0.227 0.460 0.688 0.460 0.227]

➤ Matlab

❑ Eqberdemo



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MLSE

Advantages

- Optimal
- Lower bound for other equalizers
- Can, sometimes, completely eliminate ISI

Disadvantages

- High complexity
- Output delayed
- Knowledge of channel
 - ❑ Typically use a training sequence
 - ❑ Truncate the estimated impulse response to mitigate complexity

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Taxonomy

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Equalizers

➤ Sequence

- ❑ MLSE
- ❑ Suboptimal MLSE

➤ Symbol – by – symbol (*one shot*)

- ❑ linear filtering
- ❑ feedback

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Symbol-by-symbol (*one shot*)

➤ Linear filtering

- ❑ Zero forcing equalizer (ZF)
 - Find the inverse of the frequency response of the channel
- ❑ MMSE equalizer
 - Minimize the quadratic error

➤ Feedback

- ❑ Decision Feedback Equalizer (DFE)

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Linear vs. non-linear

➤ Linear

- ❑ ZF
- ❑ MMSE

➤ Non-linear

- ❑ MLSE
- ❑ DFE

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Fixed vs. adaptive

- Static channel
 - ❑ fixed equalizer is optimal
- Dynamic channel
 - ❑ DFE
 - ❑ adaptive versions of ZF and MMSE
 - LMS – minimizes quadratic error
 - RLS – recursive solution

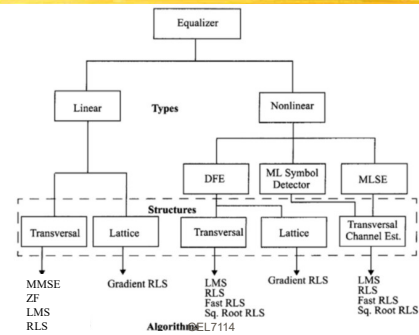
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Taxonomy



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Frequency technique

- DMT (OFDM)
 - ❑ discrete multitone
 - ❑ completely different strategy than filtering or MLSE

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Linear Equalizers

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Intersymbol Interference

- Pulse spreading after propagation through a channel with insufficient bandwidth
- spreading of the pulse by the chromatic dispersion of an optical fiber
- overlapped impulses due to multiple reflections or paths

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Multipath

- Multiple reflections
 - ISI due to reflections
 - delay spread > symbol time



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Multipath

- Impulse response
- Overlapping bits
 - Generates intersymbol interference
 - BER floor
- Example with two paths
 - time for the second path $\approx T_s$
 - second route attenuated
 - memory: one symbol
 - Four important bit patterns
 - 00 01 10 11

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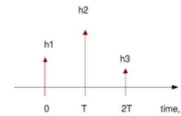
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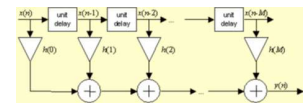
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Tapped delay line

- Model for a multipath channel
- Method to characterize the impulse response of a channel with ISI



$$h(t) = \sum_{i=1}^M h_i \delta(t - iT)$$



$$p_c(t) = x(t) * h(t) = \sum_{i=1}^M h_i x(t - iT)$$

$$p_c(nT) = \sum_{i=1}^M h_i x((n-i)T)$$

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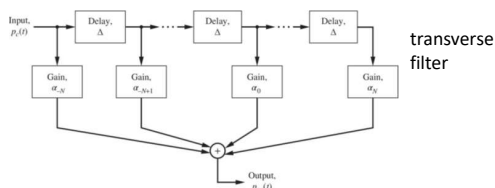
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Linear equalizer

- Filter to correct the tapped delay line channel response
- Another tapped delay line !
- Choose filter coefficients to force zero ISI



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Zero Forcing

Linear Equalizers

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Zero ISI Criterion

- Input to the equalizer p_c (après le channel)
- Output

$$p_{eq}(t) = \sum_{n=-N}^N \alpha_n p_c(t - n\Delta)$$

- Force zero ISI

$$p_{eq}(mT) = \sum_{n=-N}^N \alpha_n p_c[(m-n)T]$$

$$= \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \quad m = 0, \pm 1, \pm 2, \dots, \pm N$$

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Vector notation

$$\sum_{n=-N}^N \alpha_n p_c[(m-n)T]$$

$$[A] = \begin{bmatrix} \alpha_{-N} \\ \alpha_{-N+1} \\ \vdots \\ \alpha_N \end{bmatrix} \quad P_c A = \begin{bmatrix} \sum_{n=-N}^N \alpha_n p_c[(-N+n)T] \\ \vdots \\ \sum_{n=-N}^N \alpha_n p_c[(m-n)T] \\ \vdots \end{bmatrix}$$

$$[P_c] = \begin{bmatrix} p_c(0) & p_c(-T) & \cdots & p_c(-2NT) \\ p_c(T) & p_c(0) & \cdots & p_c[(-2N+1)T] \\ \vdots & \vdots & \ddots & \vdots \\ p_c(2NT) & p_c(NT) & \cdots & p_c(0) \end{bmatrix}$$

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Vector notation

$$P_c A = \begin{bmatrix} \sum_{n=-N}^N \alpha_n p_c [(-N+n)T] \\ \vdots \\ \sum_{n=-N}^N \alpha_n p_c [(m-n)T] \\ \vdots \end{bmatrix} \quad [P_{eq}] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\left. \begin{matrix} \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$
 $\left. \begin{matrix} 1 \\ \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$

$$p_{eq}(mT) = \sum_{n=-N}^N \alpha_n p_c [(m-n)T]$$

$$= \begin{cases} 1, & m=0 \\ 0, & m \neq 0 \end{cases} \quad m$$

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Vector notation

➤ For the right choice of $\{\alpha\}$

$$[P_{eq}] = [P_c][A]$$

$$[P_{eq}] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\left. \begin{matrix} \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$
 $\left. \begin{matrix} 1 \\ \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$

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Find $\{\alpha\}$

$$[A] = [P_c]^{-1} [P_{eq}] = [P_c]^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \text{middle column of } [P_c]^{-1}$$

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Frequency domain interpretation

➤ impulse response effective

- channel + equalizer
- Dirac delta function

➤ Effective frequency response

- channel + equalizer
- flat spectrum – constant in frequency
- all-pass filter

$$[P_{eq}] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\left. \begin{matrix} \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$
 $\left. \begin{matrix} 1 \\ \vdots \\ 0 \end{matrix} \right\} N \text{ zeros}$

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Analysis with z transform

➤ Effect in the frequency domain of the choice

$$[A] = [P_c]^{-1} [P_{eq}] = [P_c]^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- ZF filter frequency response is the inverse of the channel
- Total effect is equivalent to a FLAT channel

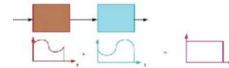
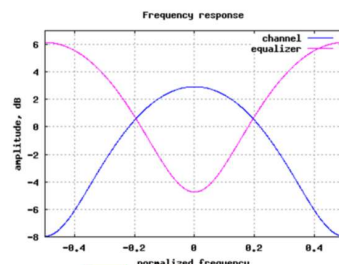


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Analysis with z transform

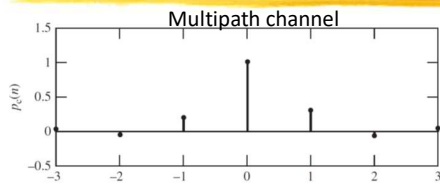


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Example



- channel knowledge required

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Complexity

- Possible to reduce complexity
- The cost is reduced performance
- Smaller reflections can be neglected

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Numerical example: Zero Forcing

Linear Equalizers

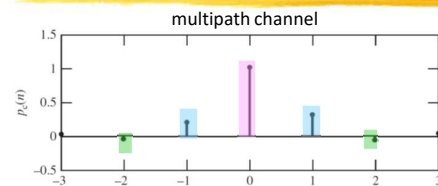
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Example



$$\begin{aligned} p_c(-3T) &= 0.02 & p_c(-2T) &= -0.05 & p_c(-T) &= 0.2 & p_c(0) &= 1.0 \\ p_c(T) &= 0.3 & p_c(2T) &= -0.07 & p_c(3T) &= 0.03 \end{aligned}$$

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Example calculation

$$[P_c] = \begin{bmatrix} 1.0 & 0.2 & -0.05 \\ 0.3 & 1.0 & 0.2 \\ -0.07 & 0.3 & 1.0 \end{bmatrix}$$

Toeplitz matrix

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$$[P_c] = \begin{bmatrix} 1.0 & 0.2 & -0.05 \\ 0.3 & 1.0 & 0.2 \\ -0.07 & 0.3 & 1.0 \end{bmatrix}$$

$$[P_c]^{-1} = \begin{bmatrix} 1.0815 & -0.2474 & 0.1035 \\ -0.3613 & 1.1465 & -0.2474 \\ 0.1841 & -0.3613 & 1.0815 \end{bmatrix}$$

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Find the filter ...

$$[A] = \begin{bmatrix} 1.0815 & -0.2474 & 0.1035 \\ -0.3613 & 1.1465 & -0.2474 \\ 0.1841 & -0.3613 & 1.0815 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.2474 \\ 1.1465 \\ -0.3613 \end{bmatrix}$$

knowledge of the channel filter coefficients

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Find the filter output

$$p_{eq}(mT) = \sum_{n=-N}^N \alpha_n p_c[(m-n)T]$$

$$p_{eq}(m) = -0.2474 p_c[(m+1)T] + 1.1465 p_c(mT) - 0.3613 p_c[(m-1)T]$$

0.2 1 0.3

+ neglected terms

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Find the filter output

$$p_{eq}(m) = -0.2474 p_c[(m+1)T] + 1.1465 p_c(mT) - 0.3613 p_c[(m-1)T]$$

neglected terms

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Performance: Zero Forcing

Linear Equalizers

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Frequency domain

$$p_{eq}(mT) = \sum_{n=-N}^N \alpha_n p_c[(m-n)T]$$

$$H_{eq}(f) = \sum_{n=-N}^N \alpha_n \exp(-j2\pi n f T)$$

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AWGN noise

- Signal bandwidth $B = 1/2T$
- Noise level after equalizer

$$\sigma_N^2 = \frac{N_0}{2T} \sum_{j=-N}^N \alpha_j^2$$

➤ BPSK $Q\left(\sqrt{\frac{1}{\sum_j \alpha_j^2} \frac{2E_b}{N_0}}\right)$

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AWGN noise

- Signal bandwidth $B=1/2T$

$$\sigma_N^2 = \frac{N_0}{2T} \sum_{j=-N}^N \alpha_j^2$$

- BPSK $Q\left(\sqrt{\frac{1}{\sum_j \alpha_j^2} \frac{2E_b}{N_0}}\right)$ signal-to-noise ratio penalty

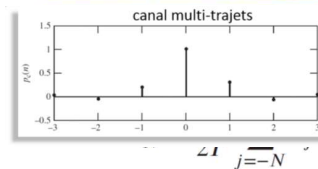
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AWGN



T pour le signal

1.78 dB noise enhancement

- BPSK $Q\left(\sqrt{\frac{1}{\sum_j \alpha_j^2} \frac{2E_b}{N_0}}\right)$

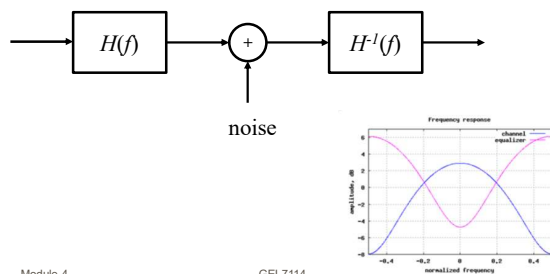
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ZF equalizer and noise

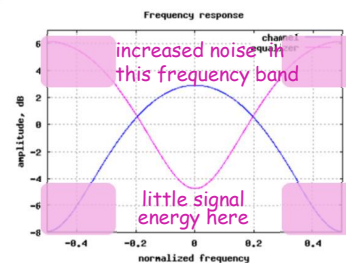


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Inverting the channel



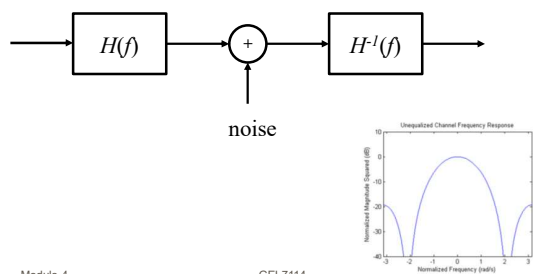
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ZF Equalizer



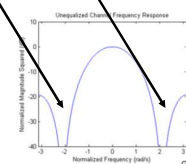
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ZF Equalizer

$[P_c]^{-1}$
it is impossible to find an inverse;
noise severely increased



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ZF Equalizer

- Simple calculation
- Simple structure
- Effective for high SNR
- Ideal equalizer when there is no noise
- Signal-to-noise penalty
- Does not exist for some channels
- Very high loss for some channels

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Examples: Zero Forcing

Linear Equalizers

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Two views of the channel

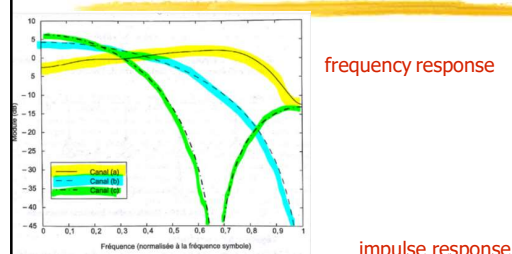


Tableau 7.1 - Réponses impulsionnelles des trois canaux de Proakis.

Canal (a)	0.04	-0.05	0.07	-0.21	-0.5	0.72	0.36	0	0.21	0.03	0.07
Canal (b)	0.407	0.815	0.407								
Canal (c)	0.227	0.44	0.688	0.44	0.227						

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Two interpretations of ZF

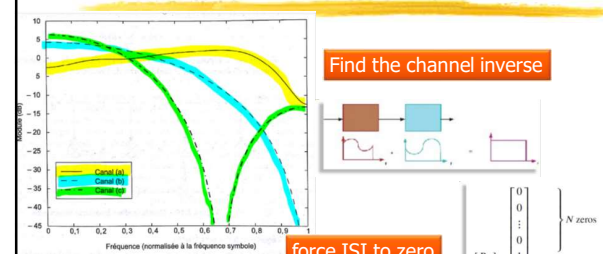


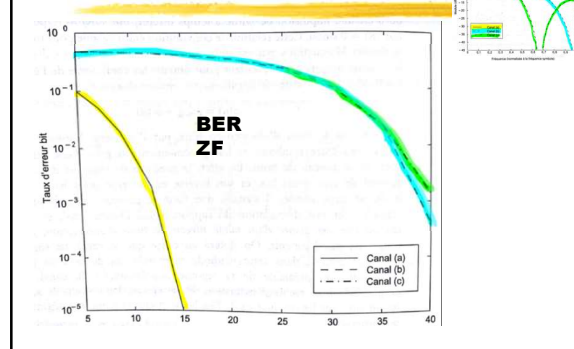
Tableau 7.1 - Réponses impulsionnelles des trois canaux de Proakis.

Canal (a)	0.04	-0.05	0.07	-0.21	-0.5	0.72	0.36	0	0.21	0.03	0.07
Canal (b)	0.407	0.815	0.407								
Canal (c)	0.227	0.44	0.688	0.44	0.227						

$$[P_{eq}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} N \text{ zeros} \\ N \text{ zeros} \end{matrix}$$

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Performance



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MMSE

Linear Equalizers

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Alternative

- Zero ISI is not a good strategy if the noise is too high
- Criterion that takes into account the accuracy of the equalizer output
- Minimum mean square error equalizer
 - ❑ MMSE equalizer

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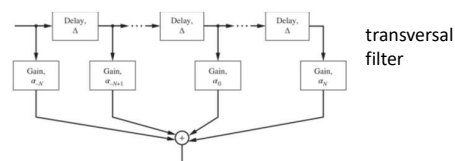
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Linear equalizer

- Same structure as ZF equalizer
- Different coefficients



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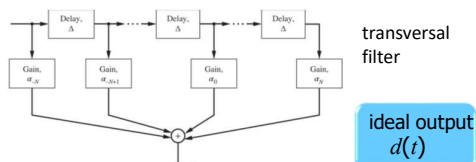
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Linear equalizer

- Same structure as ZF equalizer
- Different coefficients



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MMSE Criterion

$$E \{ [z(t) - d(t)]^2 \} = \text{minimum}$$

$$z(t) = \sum_{n=-N}^N \alpha_n y(t - n\Delta)$$

noise present

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MMSE

- Takes into account the presence of noise
- Seeks coefficients that make the output closest to the ideal signal
- Requires knowledge of data
- Transmission efficiency decreased by the use of a known data header (training sequence)

header data

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MMSE Criterion

$$E \{ [z(t) - d(t)]^2 \} = \text{minimum}$$

$$z(t) = \sum_{n=-N}^N \alpha_n y(t - n\Delta)$$

noise present

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Correlation

$$R_{yd}(\tau) = E[y(t)d(t + \tau)]$$

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Vector notation

$$R_{yd}(\tau) = E[y(t)d(t + \tau)]$$

$$[R_{yd}] = \begin{bmatrix} R_{yd}(-N\Delta) \\ R_{yd}[-(N-1)\Delta] \\ \vdots \\ R_{yd}(N\Delta) \end{bmatrix}$$

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Autocorrelation

$$R_{yy}(\tau) = E[y(t)y(t + \tau)]$$

$$[R_{yy}] = \begin{bmatrix} R_{yy}(0) & R_{yy}(\Delta) & \cdots & R_{yy}(2N\Delta) \\ R_{yy}(-\Delta) & R_{yy}(0) & \cdots & R_{yy}[(N-1)\Delta] \\ \vdots & & & \vdots \\ R_{yy}(-2N\Delta) & & \cdots & R_{yy}(0) \end{bmatrix}$$

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MMSE Solution

$$E\{[z(t) - d(t)]^2\} = \text{minimum}$$

$$[R_{yy}][A]_{\text{opt}} = [R_{yd}]$$

Wiener-Hopf
Equations

$$[A]_{\text{opt}} = [R_{yy}]^{-1}[R_{yd}]$$

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Examples : MMSE

Linear Equalizers

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Calculation

➤ Also needs to invert a matrix

□ MMSE: correlation matrixes

□ ZF: impulse response matrix

➤ Minimal error

$$\sigma_d^2 - [R_{yd}]^T [A]_{\text{opt}}$$

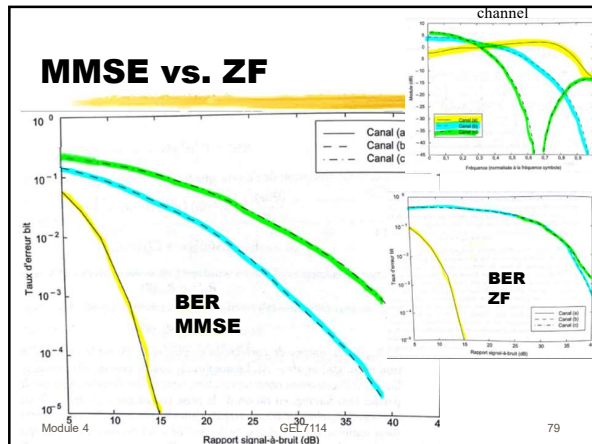
$$\sigma_d^2 = E[d^2(t)]$$

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Equalizer options

- Time between samples $z(t) = \sum_{n=-N}^N \alpha_n y(t - n\Delta)$
- Typically $\Delta = T$ symbol time
 - good compromise complexity-improvement
- Possible to have $\Delta = T/n$
 - fractionally spaced equalizer
 - good for reflections with delay $< T$
 - good when the clock is inaccurate (presence of jitter)

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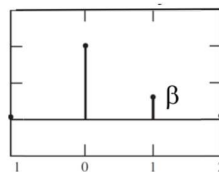
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Another example

multipath channel



- channel knowledge required

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MMSE filter

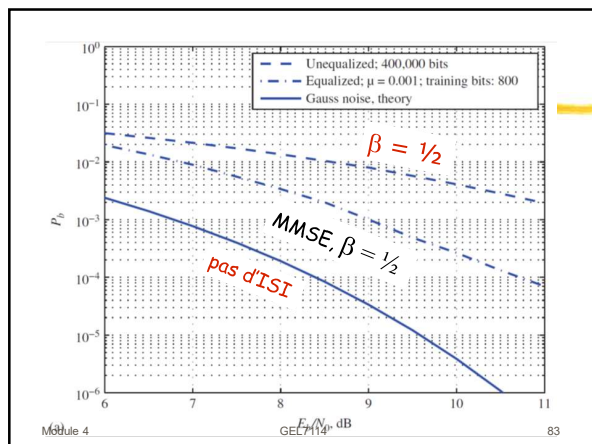
- Coefficients $\begin{bmatrix} 2\beta \frac{E_b}{N_0} \\ 2\frac{E_b}{N_0} \\ 0 \end{bmatrix}$
- $\beta = 1/2$ and $E_b/N_0 = 10$ dB $\begin{bmatrix} 0.045 \\ 0.882 \\ -0.339 \end{bmatrix}$

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MMSE equalizer

- Simple structure
- Calculating correlation matrix
- Exists for all channels
- ZF equivalent for high SNR
- Signal-to-noise penalty

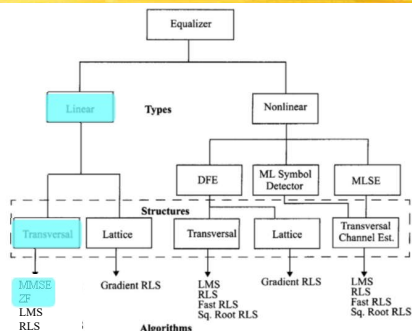
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Taxonomy



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Adaptative version

Linear Equalizers

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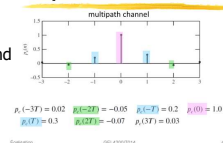
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Channel knowledge

Series de measurements

- explicit channel estimate
- impulse response coefficients found
- good for static channels

Example



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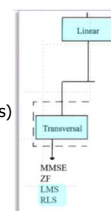
Channel knowledge

Series de measurements

- explicit channel estimate
- impulse response coefficients found
- good for static channels

Filter adaptation

- implicit channel estimate
- filter coefficients found by adaptation (not manipulation of impulse response coefficients)
- Known data needed for adaptation
- data-aided vs. decision-directed



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"Data-aided" adaptation

Exploit a header to look for adaptive coefficients



Essentially a technique to estimate the channel

Important for channels that vary in time

- The coefficients must change with the channel...



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too inefficient for a fast-changing channel

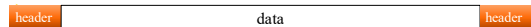
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Decision-directed adaptation

Decision-directed

Solution for updating coefficients



Between arrivals of a header

- coefficient adaptation proceeds, but using the receiver's decisions as "known data"
 - May be unstable
- Vulnerable to a total loss of communications (outage)

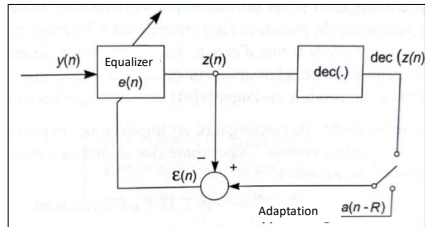
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Decision-directed adaptation



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Decision Feedback (DFE)

Non-linear equalizer

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Decision Feedback Equalizer

- Feedback that makes the filter non-linear
- Two filters at the receiver
 - ❑ Adaptive filtering for equalizer
 - ❑ Adaptive filtering to recreate the ISI
 - ❑ Recreated ISI is exploited by the equalizer
- Distinction between decision-directed and decision-feedback
 - ❑ Decision-directed
 - RX decisions used for adaptation (as known data)
 - ❑ Decision-feedback
 - Decisions used to recreate and subtract ISI from the received signal
 - Equalizer adaptation done with the majority of ISI removed
 - More efficient for channels with severe distortion
 - Effective even for channels with frequency nulls

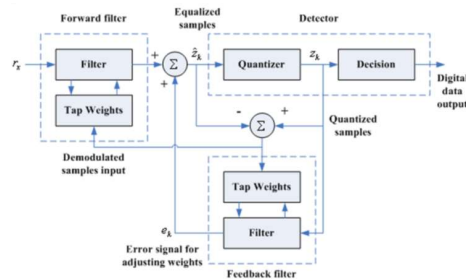
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Decision Feedback Equalizer



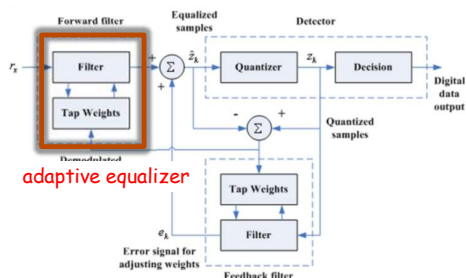
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Decision Feedback Equalizer



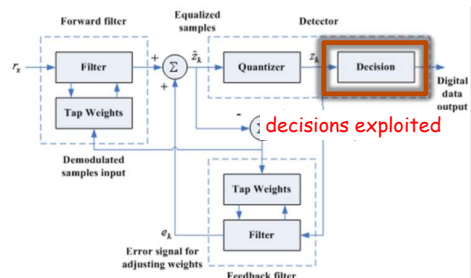
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Decision Feedback Equalizer



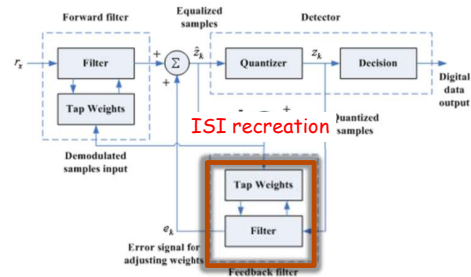
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Decision Feedback Equalizer



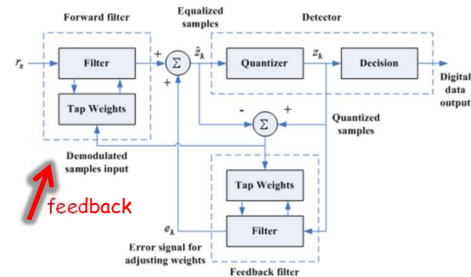
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Decision Feedback Equalizer



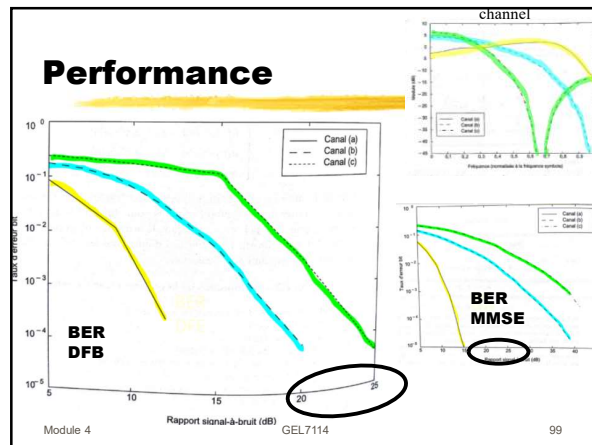
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Performance



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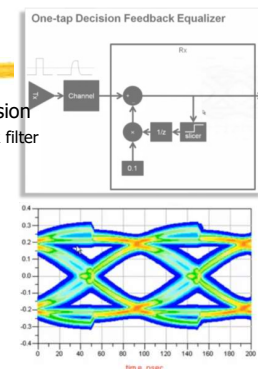
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Example

- Simple and low complexity version
 - ❑ Only one element in the feedback filter
 - ❑ No equalizer
 - ❑ "one-tap equalizer"
- Eye diagram visualization

<http://signal-integrity.blogspot.com/2012/03/decision-feedback-equalizer/decision-feedback-equalizer.html>



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OFDM

Orthogonal frequency Division Multiplexing

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Comparison

- FDM frequency multiplexing
 - ❑ Typically for many users
 - ❑ Physically separated oscillators
 - ❑ Non-coherent detection
- OFDM
 - ❑ Typically for a single user
 - ❑ Different symbol stream on each frequency (but not different users)
 - ❑ Coherent detection

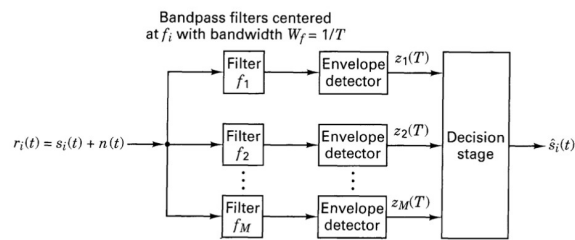
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Practical FSK / FDM



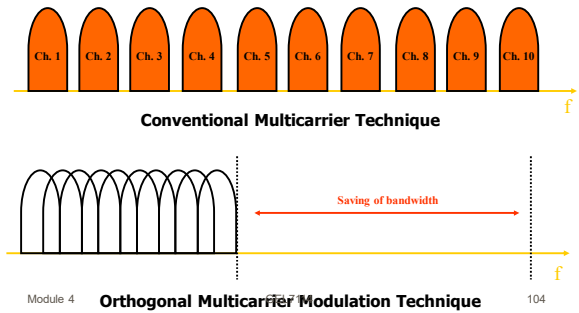
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FDM vs. OFDM



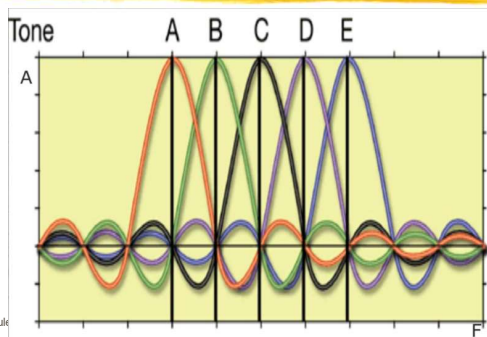
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Orthogonal Multicarrier Modulation Technique

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OFDM Spectrum

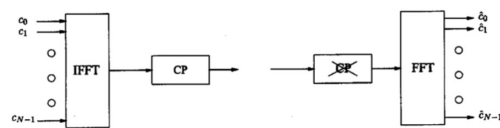


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FFT – key component

- High-performance and inexpensive DSP



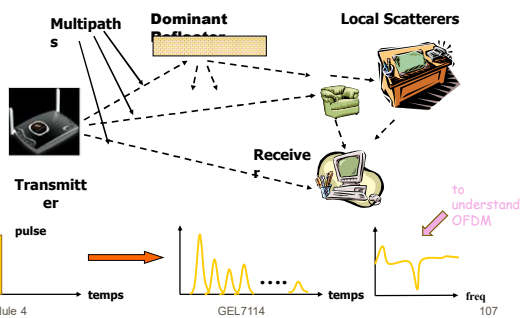
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Wireless channel



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OFDM

Principles of operation

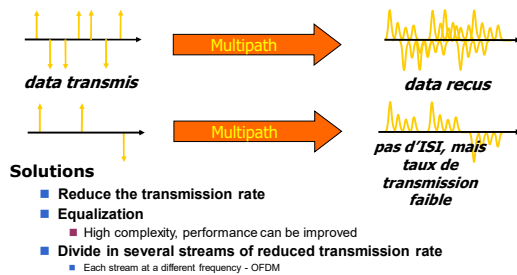
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Intersymbol Interference (ISI)



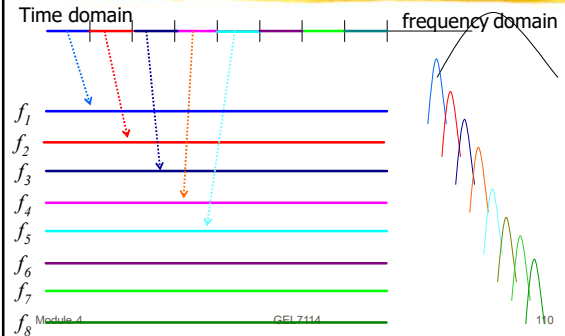
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Expand the symbol interval



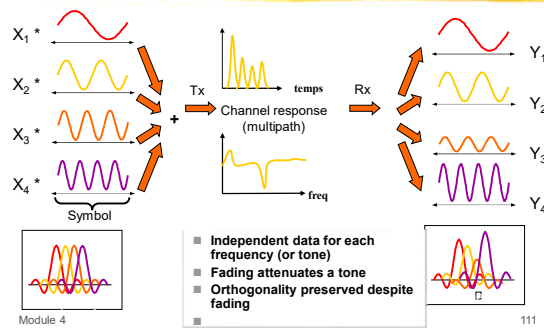
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OFDM Modulation



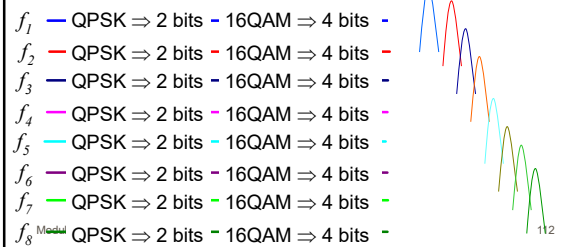
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Advanced modulations

- BPSK, QPSK, 16QAM, 64QAM
- Same bandwidth, higher binary rate for large M



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OFDM

- Divided signal frequency band
 - non-overlapping, independent sub-bands
 - independent data for each sub-band
- Spectrally effective (orthogonality)
- Efficient implementation (FFT)
- Compatible with QAM, M-PSK
- Flexible - modifiable constellations
- Avoids equalization

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Mathematical manipulations

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FFT – key component

- High-performance and inexpensive DSP



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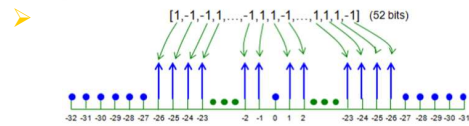
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Example: BPSK

- Data



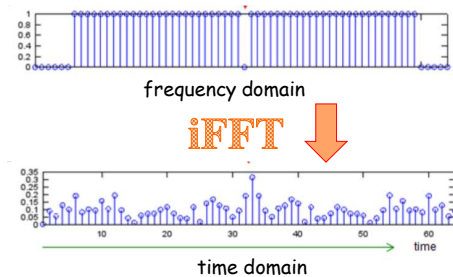
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Role of the IFFT



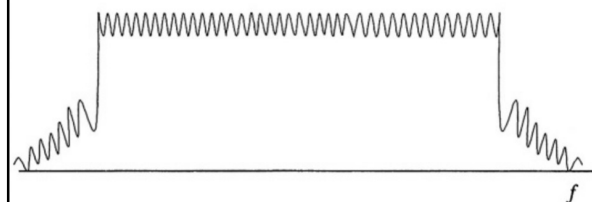
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OFDM Spectrum



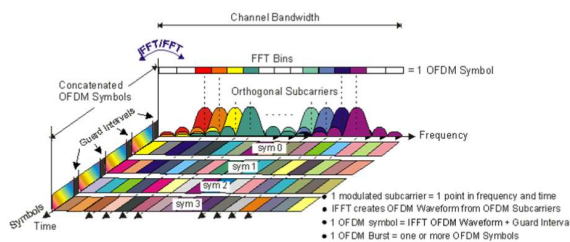
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OFDM time et frequency



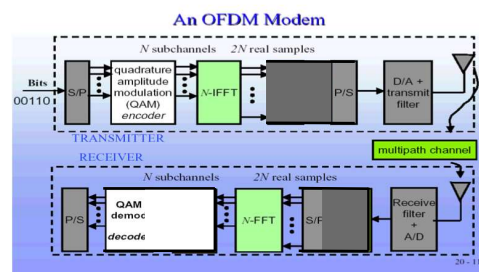
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OFDM Implementation



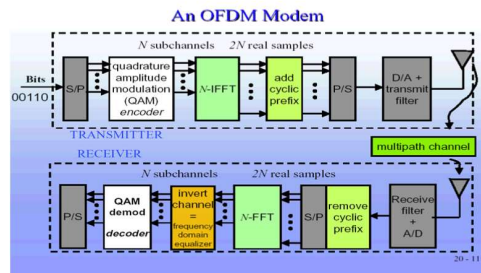
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OFDM Implementation



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One-tap equalizer

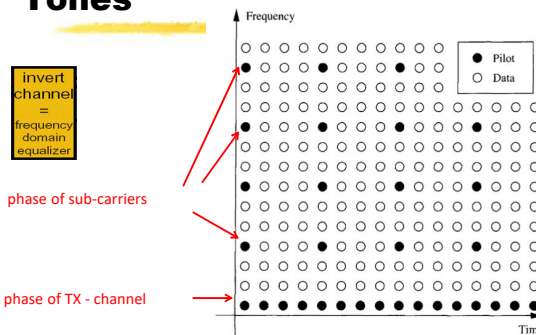
- OFDM with IFFT
 - One oscillator at TX
 - Only one phase to track
- Channel effects

phase may vary with sub carrier
* coherent detection *

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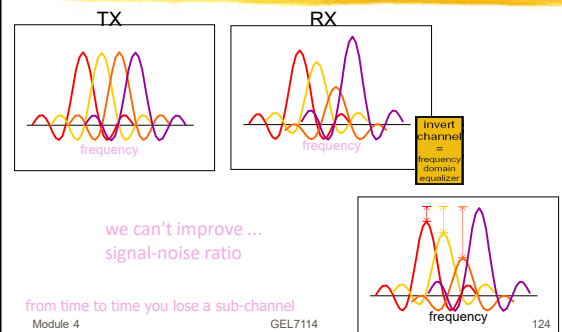
Tones



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OFDM Modulation



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FFT – key component

- High-performance and inexpensive DSP
- Compromise in the number of subcarriers
 - more carriers, more complex
 - the more carriers, the more ideal the sub-band is



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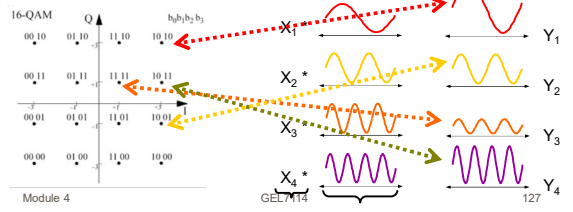
Peak-to-Average

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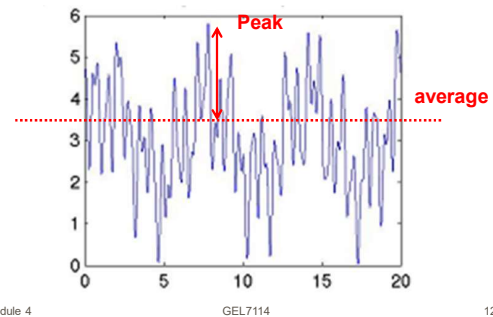
QAM & OFDM Modulation

- QAM - variable symbol energy
- OFDM – sum of sinusoids



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Signal in time



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OFDM Summary

- Advantages of OFDM
 - ❑ Robust against multipath
 - ❑ Less complex than equalization
 - ❑ Bandwidth efficient
 - ❑ Robust against narrowband interference
- Disadvantages of OFDM
 - ❑
 - ❑
 - ❑
 - ❑

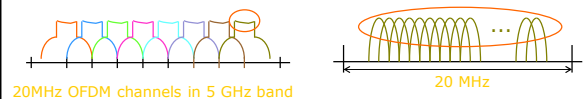
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WiFi Standard



- 802.11a
 - ❑ ISM band (5 GHz, but also 2.4 GHz)
 - ❑ Several channels, each 20 MHz
 - ❑ Guard band to avoid interference

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Discrete multitone
DMT

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Optimization of OFDM

- OFDM
 - ❑ Same power for each subcarrier
 - ❑ Same constellation for each subcarrier
- DMT (Discrete multitone)
 - ❑ Optimize power and modulation choice for each subcarrier
 - ❑ Maximize channel capacity

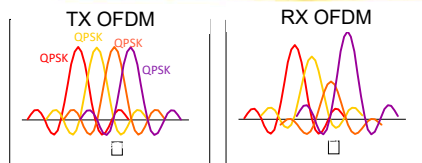
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OFDM Modulation



optimize the
choice of
modulation

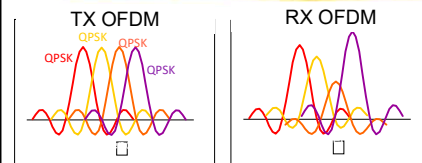
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DMT Strategy



optimize the
choice of
modulation

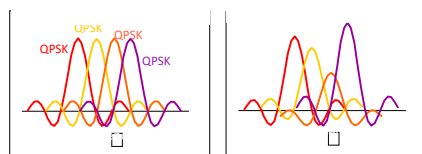
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Another DMT strategy



optimize power
choice per
subcarrier

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Two optimizations

- For linear channels, but not flat
- Choice 1
 - Probability of error P_b fixed
 - Find the maximum data rate subject to P_b
- Choice 2
 - Data rate R_b fixed
 - Find the probability of minimum error to reach R_b

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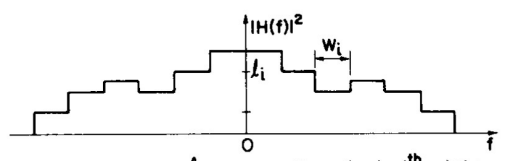
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Maximizing the data rate R_b

- Choice 1
 - Probability of error P_b fixed
 - Find the maximum data rate subject to P_b

- Narrow spacing
 - Flat channel for a subcarrier
 - Each channel at the receiver
 - Response H_j & Noise N_j



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Subcarrier modulation

- n_i bits in the constellation for carrier i
- Transmission rate $R_b = \sum_{i=1}^N n_i W_i$ bits/s
- Total power divided & allocated to subcarriers
 - Power P_j
$$\sum_j P_j = P$$
- Find $\{P_j\}$ that **maximizes R_b** and **meets error target**

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Solution

- Seek an equal probability of error for all carriers
- Channel frequency response is non-flat
- Noise is flat



signal to noise ratio
varies across subcarriers

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Solution

- Let's flip the frequency response upside down
- Now we can think of the noise-to-signal ratio

$$N(f)/|H(f)|^2$$



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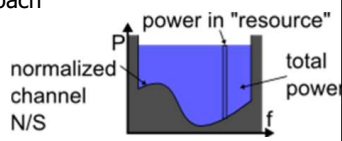
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Solution

- Let's flip the frequency response upside down
- Now we can think of the noise-to-signal ratio

$$N(f)/|H(f)|^2$$

- "waterfilling" approach



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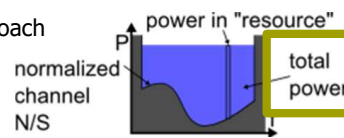
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Solution

- Subcarriers with less signal strength get more power
- Keep pouring until out of water
- Some subcarriers may get no power

- "waterfilling" approach



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Two optimizations

- The channel is linear, but not flat
- -
 -
- Choice 2
 - Data rate R_b fixed
 - Find the probability of minimum error to reach R_b

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Subcarrier modulation

- n_i bits in the constellation for carrier i
- **Fixed total rate** $R_b = \sum_{i=1}^N n_i W_i$ bits/s
- Total power divided & allocated to subcarriers
 - Power P_j $\sum_j P_j = P$
- Find the $\{P_j\}$ that meets R_b and **minimizes error**

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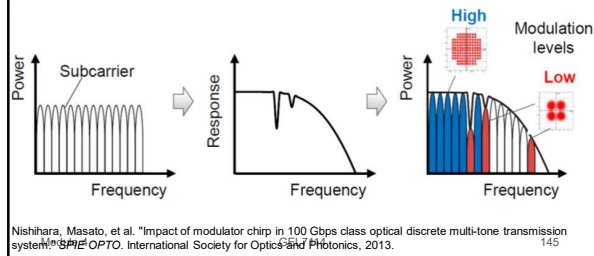
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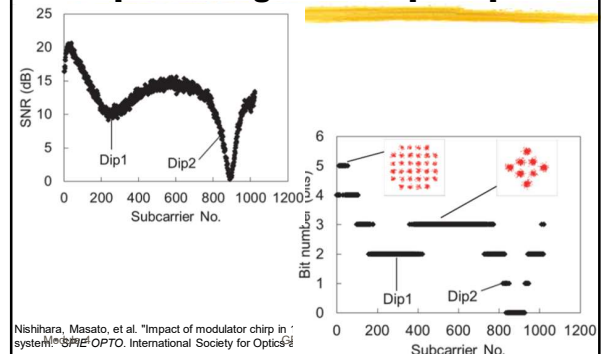
Example

➤ Allocation BITS and POWER



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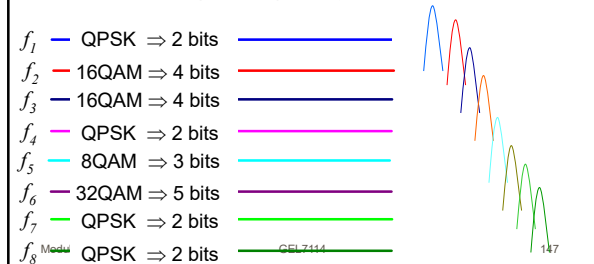
Compensating the Freq Resp



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DMT approach

- BPSK, QPSK, 16QAM, etc.
- variable binary rate depending on the channel



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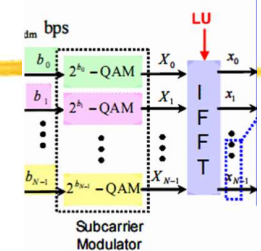
OFDM vs. DMT

➤ OFDM

- Equal powers
- Equal # bits

➤ DMT

- Adapted to the channel
- Power allocated per subcarrier
- Bits allocated per subcarrier



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Questions on comprehension

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- Contrast the ZF and MMSE equalizers in terms of
 - structure and implementation,
 - existence of an adaptive version, and
 - use of channel knowledge and/or training sequence.

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- Explain how channel properties influence the structure of the ZF and the MMSE equalizer. That is, through what mechanism does a ZF equalizer change for one channel versus another? Same question for MMSE equalizer.

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What is the optimal equalizer for a channel with intersymbol interference?

- What is the optimal equalizer for a channel with intersymbol interference?

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- Consider a modulation with M symbols in the constellation. Suppose the channel has a memory of L symbols. How does complexity of the brute force calculation of the optimal equalizer grow? How does the Viterbi algorithm complexity compare to the brute force calculation of the optimal equalizer?

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- Describe how the ZF equalizer varies for a static versus a dynamic channel.
- Suppose 1,000 symbols are needed to estimate the channel. Propose a strategy for BPSK 1 kb/s transmission using a ZF equalizer over a channel that is
 - stable and unchanging on an hourly basis,
 - changes every .1 sec,
 - changes every .01 sec,
 - changes every .0001 sec

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- Describe the difference between a data-aided versus a decision-directed adaptive MSME equalizer. For the following respond a) data-aided, b) decision-directed, or c) not practical. 16QAM modulation for 1 kbs over a channel that
 - changes every .1 sec,
 - changes every .01 sec,
 - changes every .0001 sec

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- Label the following curves
 - ☐ the minimum mean squared error equalizer,
 - ☐ theoretical BER without ISI,
 - ☐ the ZF equalizer,
 - ☐ the maximum likelihood sequence estimator

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- What frequency domain features of the channel would make a zero forcing equalizer less attractive than other equalizers? Justify your response.

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- Give a sketch of the block diagram of a decision-directed zero forcing equalizer.

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- Explain how DMT and OFDM differ and how they are similar.

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- What are the two parameters that can be varied for each subcarrier in DMT?

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- Explain the waterfilling analogy for DMT.

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- How does the number of subcarriers affect the performance of DMT or OFDM?

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- Why does DMT seek to have the same BER on all subcarriers?

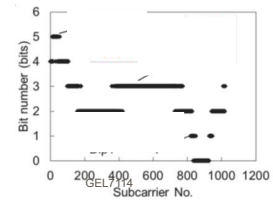
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- Interpret the following plot of the bit allocation of a DMT solution.
- Give a sketch of the likely frequency response that led to this bit allocation.



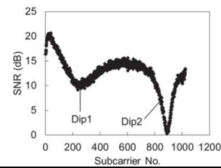
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Which would be more likely to have BPSK assigned? Defend your answer.

- Which of the following subcarriers would be more likely to be assigned 16QAM in a DMT system?
- Which would be more likely to have BPSK assigned?
- Would the weakest carrier be more likely to
 - ☐ have the most power assigned to it,
 - ☐ have zero power assigned to it, or
 - ☐ could have either zero, the most or another value of power assigned to it.
- Justify your response.



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- How does OFDM avoid ISI?

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- How is VLSI used in an OFDM transceiver?

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- Assuming the same transmission rate, why is OFDM better than BPSK for a multipath channel?

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What is the order of
OFDM processes :

- ☐ Adding cyclic prefix
- ☐ Decoding
- ☐ Demodulation
- ☐ Encoding
- ☐ FFT
- ☐ IFFT
- ☐ Modulation
- ☐ Removing cyclic prefix

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