

This week ...

- Topic
 - ❑ Mathematical modeling of noise (e.g., Monte Carlo simulation, simulation of random Poisson and Weiner processes, simulation of electrical and optical noise)
 - ❑ Optical noise in context of EDFAs in WDM systems
- Objectives
 - ❑ Develop analytical equations to describe BER performance due to PIN noise and ASE noise
 - ❑ Understand impact of filtering (optical and electrical) on BER performance
 - ❑ Simulate PIN noise, ASE noise and filtering effects
 - ❑ Develop analytical model for Poisson and Weiner processes as a limit of sum processes
 - ❑ Simulate Poisson and Weiner processes

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Resources

- Agrawal, G. P., *Fiber-Optic Communication Systems*, Chapter 4 Optical Receivers, Wiley, fourth edition, 2010.
- Olsson, N. A., "Lightwave Systems with Optical Amplifiers," *Journal of Lightwave Technology*, vol. 7, no. 7, pp. 1071-1082, Jul 1989.
- Winzer, P.J.; Pfennigbauer, M.; Strasser, M.M.; Leeb, W.R.; , "Optimum filter bandwidths for optically preamplified NRZ receivers," *Journal of Lightwave Technology*, vol. 19, no. 9, pp. 1263-1273, Sep 2001
- Leon-Garcia, A., *Probability, statistics, and random processes for electrical engineering*, Chapter 9 Random Processes, Pearson/Prentice Hall, 4th edition, 2008.
- Source code for generating Poisson and Weiner processes in Matlab
- Homework assignment – due 28 January 2013

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Noise sources

- Quantification noise
- Synchronization errors (*jitter*)
- Channel noise
 - ❑ thermal noise
 - ❑ interference (crosstalk)
- Intersymbol Interference (ISI)
 - ❑ dispersion
 - ❑ channelization
 - ❑ patterning effects (gain saturation)

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PIN and ASE noise

- We now derive the BER expressions for IM/DD systems with a PIN photodiode
- ... and ASE !

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PIN detection

January-16-12
3:51 PM

Other weeks will cover components, this week I want to focus on noise at the receiver, so we will be focusing on these elements

- 1) filters
- 2) PD (PIN)
- 3) ASE

PIN is basically a square law device. What ever signal is input (constant or time varying) will see it field squared during the optical to electrical conversion. Today I will discuss direct detection - a single PIN - and will leave the subject of coherent detection (balanced PD and local oscillator, polarization multiplexing) to later weeks.

In addition to the electrical output proportional to the square of the input signal field, the PIN adds noise. One source of noise is the thermal noise we find in all electrical receivers. This noise is classical additive white Gaussian noise (AWGN) whose power spectral density is given by

$$N_{th} = \frac{2k_B T}{R_L}$$

load resistor

Boltzmann

$$S_{th}(f) = N_{th} \quad \forall \quad f < 1/4B$$

Given an electrical filter at the receiver w/ frequency response $H_e(f)$ with noise equivalent bandwidth of BW_{eq} the sampled noise will be zero mean Gaussian with variance

$$\sigma_{th}^2 = N_{th} \cdot BW_{eq}$$

↑
density of
thermal noise
↑
bandwidth of
noise "let in" by receiver

For a Nyquist filter, the noise samples will be uncorrelated. In addition to the typical electrical thermal noise, a PIN also has shot noise due to the arrival of photons with Poissonian statistics. The shot noise is also a white process as is the thermal noise. Strictly speaking the process is Poissonian, but can be modeled as Gaussian in most telecom systems. The important distinction between thermal noise & shot noise is the the level of the PSD for shot noise **varies with the input power**. In the case of digital modulation with OOK (intensity modulation) the signal power is different during a mark & a space — hence a logical 1 will see more shot noise than a logical 0. This signal dependent noise is not feared

in RF communications systems. Using an analysis where the BW of the PD is assumed much greater than the bit rate, we assume the shot noise is proportional to the "instantaneous" value of the input power for the mark or space.

$$N_{sn} = g \cdot R_d \cdot P_{in}$$

electron charge
responsivity $\frac{q}{h\nu}$

$$N_{sno} = g R_d P_o$$

$$N_{sni} = g R_d P_i$$

$$S_{sn}(f) = N_{sn} \quad \forall f \quad (\text{white})$$

The noise sample of the shot noise process will be

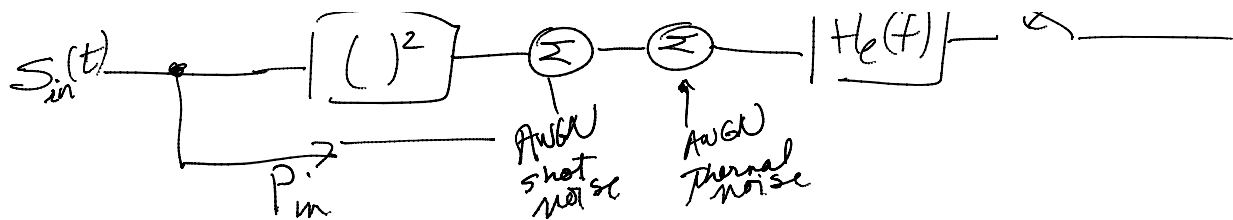
$$\sigma_{sn}^2 = N_{sn} \cdot BW_{eq}$$

as with the thermal noise, but different for the 1 and the 0 ...

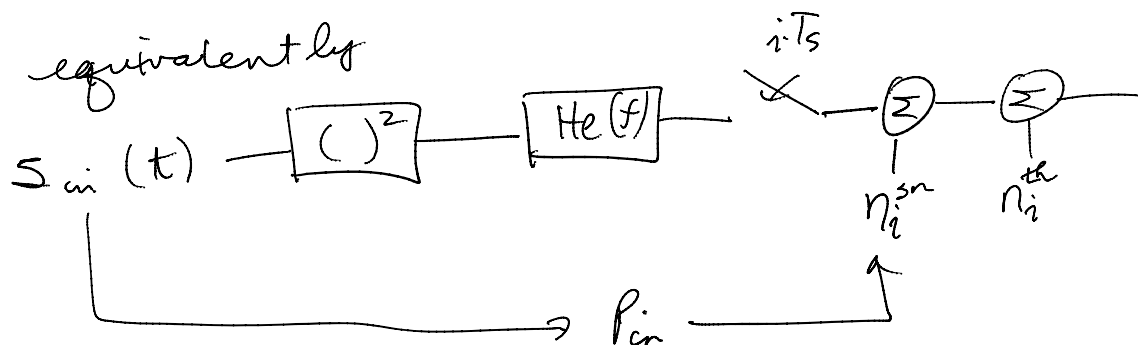
Later we will discuss how to generate a Poisson process, however for the PIN shot noise we will assume it is AWGN whose PSD is a function of the input power (time varying)

Now we know what noise is generated by the P/N. In a numerical simulator we could





or equivalently



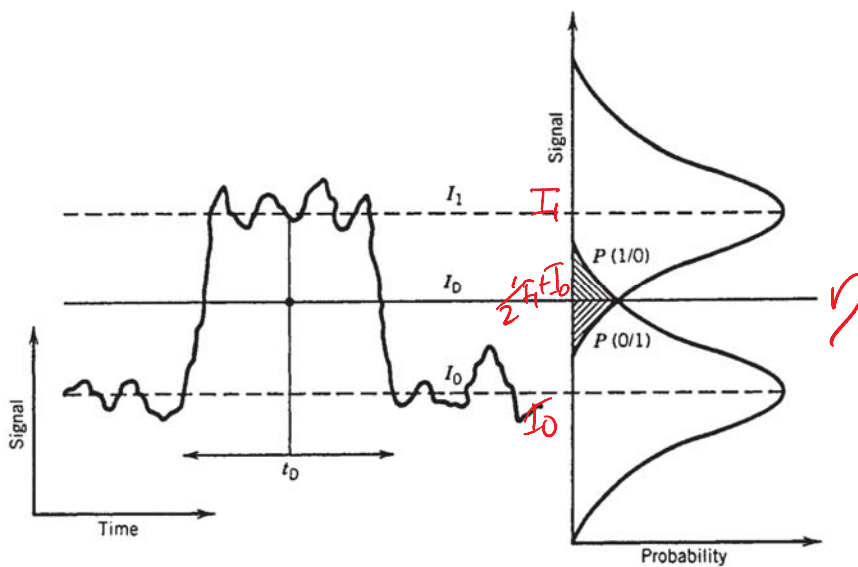
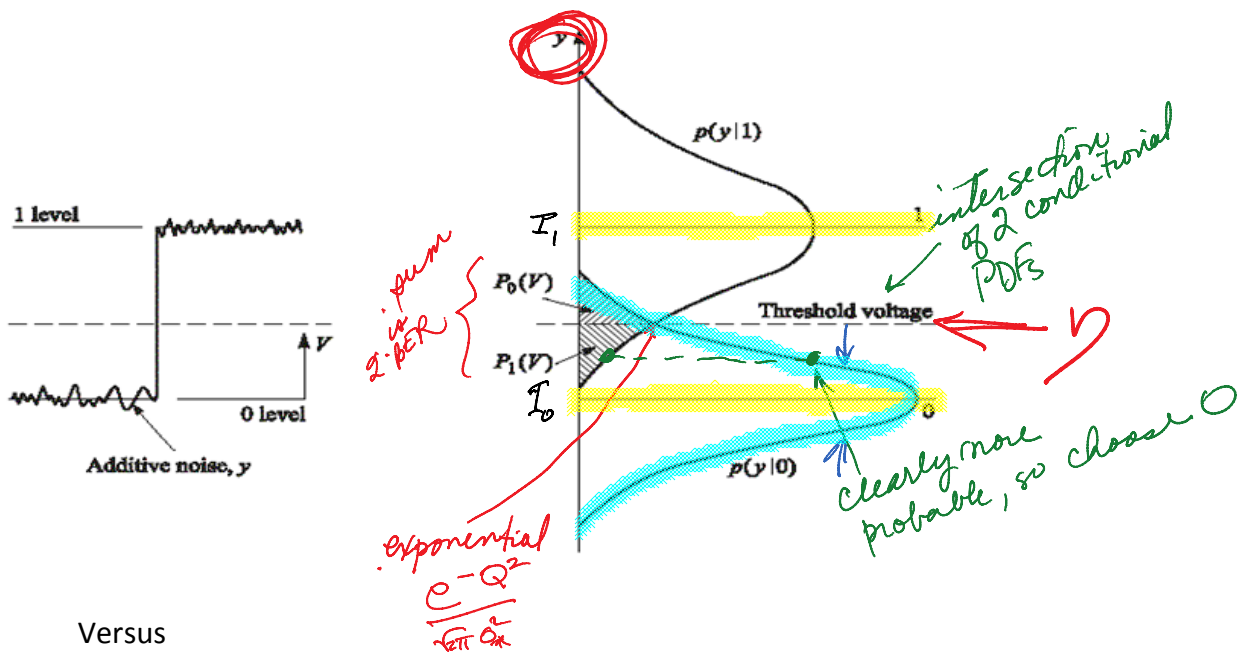
Let's suppose that we are using OOK and the signal is a monochromatic ideal laser w/zero phase noise (zero linewidth). There is no intensity noise or ASE, just a signal whose power for a logical 1 is P_1 . The OOK is implemented either by direct modulated or external modulation, but in either case has a finite extinction ratio of R_{EX}

$$P_0 = R_{EX} \cdot P_1 < P_1$$

Let $I_1 = R_d P_1$ $I_0 = R_d P_0$

be the photocurrent when sending a logical "1" or "0"

Since is Gaussian, adding a signal will simply displace the mean from zero to the corresponding photocurrent. The mean is different for a logical "1" and a logical "0". Unlike RF systems the variance of the noise is also different...



Let's calculate the bit error rate

$$\begin{aligned} \text{BER} &= p(1 \text{ sent}) \cdot p(\text{error} | 1 \text{ sent}) + p(0 \text{ sent}) \cdot p(\text{error} | 0 \text{ sent}) \\ &= \frac{1}{2} P(\text{error} | 1) + \frac{1}{2} P(\text{error} | 0) \end{aligned}$$

When do you make an error? you detector

takes the sample & compare it to a threshold η_{th}
 $\dots > \eta_{th}$ choose "1", less than η_{th} choose "0"

$$BER = \frac{1}{2} P(x < \eta_{th} | 1) + \frac{1}{2} P(x > \eta_{th} | 0)$$

conditional pdf | 1
 $x \sim \mathcal{N}(I_1, \sigma_1^2)$

cond pdf | 0 is
 $x \sim \mathcal{N}(I_0, \sigma_0^2)$

$$P(x < \eta_{th} | 1)$$

$$\int_{-\infty}^{\eta_{th}} \mathcal{N}(I_1, \sigma_1^2)$$

$$\int_{\eta_{th}}^{\infty} \mathcal{N}(I_0, \sigma_0^2)$$

What η_{th} is optimal?

intersection of two conditional pdfs...

$$\frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(\eta - I_1)^2}{2\sigma_1^2}} = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(\eta - I_0)^2}{2\sigma_0^2}} \rightarrow \frac{\sigma_1}{\sigma_0} = e^{-\frac{(\eta - I_0)^2}{2\sigma_0^2} + \frac{(\eta - I_1)^2}{2\sigma_1^2}}$$

$$\ln \frac{\sigma_1}{\sigma_0} = \frac{(\eta - I_0)^2}{2\sigma_0^2} - \frac{(\eta - I_1)^2}{2\sigma_1^2} \quad \text{or}$$

$$\frac{(\eta_{th} - I_0)^2}{2\sigma_0^2} = \frac{(I_1 - \eta_{th})^2}{2\sigma_1^2} + \ln \frac{\sigma_1}{\sigma_0}$$

η_{th} solving this equation gives the optimal threshold.

When $\sigma_1 = \sigma_0$ the $\ln = 0$ & η_{th} is simply the mid point

For $\frac{1}{2} \ln P_1/P_0 = \ln \text{extinction ratio} = \ln(20dB) = 4.6$ small compared to other terms, so try

$$\frac{(\eta I_0)^2}{2\sigma_0^2} = \frac{(I_1 - \eta)^2}{2\sigma_1^2}$$

$$\frac{\eta - I_0}{\sigma_0} = \frac{I_1 - \eta}{\sigma_1}$$

or $\boxed{\eta = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}}$

Define the Q factor by

$$\begin{aligned} \frac{\eta - I_0}{\sigma_0} &= \frac{I_1 - \eta}{\sigma_1} = \frac{I_1 - \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}}{\sigma_1} \\ &= \frac{\cancel{\sigma_0 I_1} + \sigma_1 I_1 - \cancel{\sigma_0 I_1} - \sigma_1 I_0}{\sigma_1 (\sigma_0 + \sigma_1)} = \frac{I_1 - I_0}{\sigma_1} \cdot \frac{\sigma_1}{\sigma_0 + \sigma_1} = \frac{I_1 - I_0}{\sigma_0 + \sigma_1} \end{aligned}$$

$$Q = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$$

This is the point of intersection

i.e. $e^{-\frac{(x-\mu)^2}{2\sigma^2}} = e^{-Q^2}$

$$P(\text{error}|0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(y-I_0)^2}{2\sigma_0^2}} dy \quad \begin{aligned} &\stackrel{\text{erfc}}{=} \frac{\eta - I_0}{\sqrt{2}\sigma_0} \\ &\stackrel{\text{erfc}}{=} \frac{Q}{\sqrt{2}} \end{aligned}$$

Since $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$

similarly

$$P(\text{error}|1) = \frac{1}{2} \text{erfc} \frac{Q}{\sqrt{2}}$$

we see that in neglecting $\ln \frac{\sigma_1}{\sigma_0}$, we

essentially found $\eta \neq 1$ $P(\text{error}|1) = P(\text{error}|0)$

In any case

$$BER = \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \quad Q = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$$

Numerically we have the approximation

$$BER \approx \frac{e^{-Q^2/2}}{Q\sqrt{2\pi}}$$

Q Factor, S/N Ratio, and BER

$$Q = \frac{V_{on} - V_{off}}{\sigma_{on} + \sigma_{off}} = \frac{1}{2} \sqrt{\frac{\text{Signal}}{\text{Noise}}}$$

$$BER = \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

For our BER of 10^{-9} , $Q=6$ and $S/N=144$

Contrast the results w/ classic OOK w/ $d_0=0$

$$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} - \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) = \frac{1}{2} - \frac{1}{2} (1 - \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right))$$

$$= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\text{or } \sqrt{\frac{E_b}{N_0}} = Q \quad \text{recall } SNR = 2 \frac{E_b}{N_0}$$

$$\text{makes sense since } Q \text{ for } d_0=0 \text{ gives } \frac{I_1 - I_0}{2\sigma} = \frac{1}{2} \frac{I_1 - I_0}{\sigma} \uparrow \sqrt{SNR}$$

So far the input optical signal was an ideal laser w/ zero intensity noise & zero phase, so the photocurrent was "fixed" during a bit and the only noise was P/N noise

Now we are going to assume some optical amplification is present & that the amplifier adds wide band incoherent optical noise referred to as ASE - amplified spontaneous emission

actually $I_i \neq I_o$
are photocurrents
at sampling times

An optical signal undergoes amplification and sees gain G and a noise process $n_{sp}(t)$ so the optical output is

$$G s(t) + n_{sp}(t)$$

The $n_{sp}(t)$ is an AWGN process with flat PSD over the signal band. An optical filter effects both signal & noise (helpfully limiting the ASE and selecting the proper signal channel). The photo diode produces photo current proportional to the square of the sum of the filtered signal and ASE field. Also the shot noise is "jazzed up" by the sum of signal + ASE power. Let's see the PD response to signal + ASE

$$[s(t) + n_{sp}(t)]^2 = s^2(t) + 2s(t)n_{sp}(t) + n_{sp}^2(t)$$

signal-ASE
beating

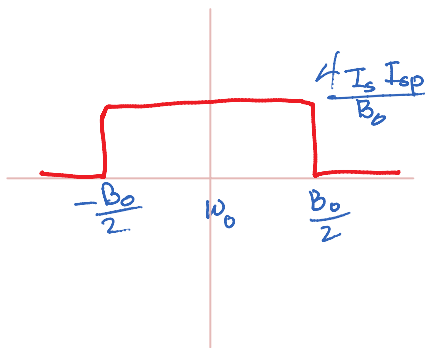
ASE-ASE beating
"sp-sp beating"

See Olsson Appendix A for the analysis of the terms.
 The analysis assumes the ASE noise is an ($\rightarrow \infty$) collection of narrowband signals ($\omega \rightarrow 0$) with a random phase & constant amplitude. The PD "averages" over the random phases when producing the current due to ASE effects

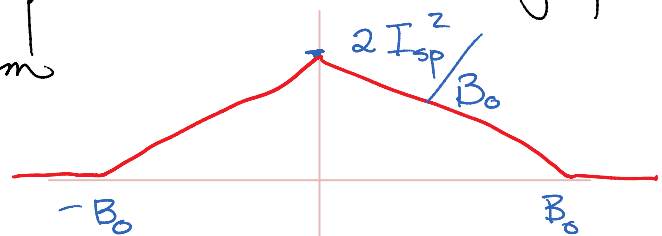
The signal-spontaneous beating will have a flat spectrum as wide as the optical filter BW with PSD equal to

$$\frac{4I_s \cdot I_{sp}}{B_0}$$

$I_s = \text{signal induced photocurrent} = R_p P_s G$
 $I_{sp} = \text{ASE} \quad \quad \quad = R_p P_{ASE}$
 $I_{sp} = \boxed{g N_{sp} (G-1) B_0}$
 range 1.4 to 4
 homework uses -35dBm/0.1nm



The spontaneous-spontaneous beating produces a triangular spectrum



Whose BW is $2B_0$ & peak is at $2I_{sp}^2/B_0$

These two spectra represent additional receiver noise due to ASE

After photo detection the electrical signal is further filtered, reducing the total noise contribution

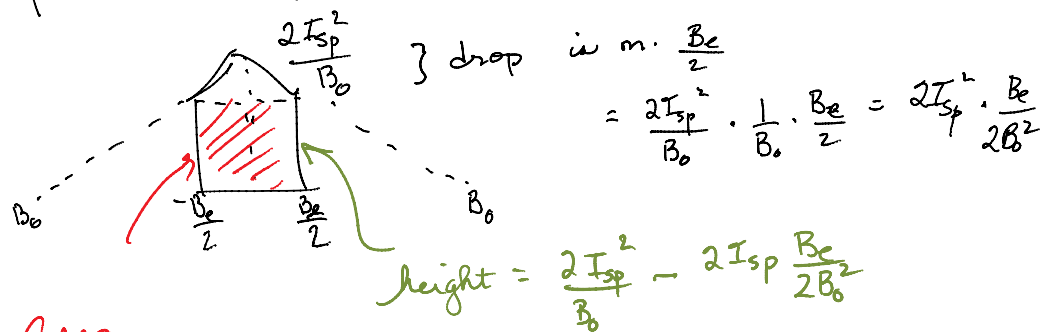
Let $B_e = BW_{eq}$ of the electrical filter

Clearly for $s-sp$ the noise variance becomes

$$\int_{-\infty}^{\infty} S_{s-sp}(f) |H_e(f)|^2 df = \int_{-\frac{B_0}{2}}^{\frac{B_0}{2}} \frac{4 I_s I_{sp}}{B_0} |H_e(f)|^2 df$$

$$\sigma_{s-sp}^2 = \frac{4 I_s I_{sp}}{B_0} \int_{-\frac{B_0}{2}}^{\frac{B_0}{2}} |H_e(f)|^2 df = \frac{4 I_s I_{sp}}{B_0} \cdot B_e$$

For the spontaneous-spontaneous



Area

$$\sigma_{sp-sp}^2 = B_e \cdot \text{height} = 2 I_{sp}^2 \cdot \frac{B_e}{B_0^2} \left(B_0 - \frac{B_e}{2} \right) = I_{sp}^2 \frac{B_e}{B_0^2} (2 B_0 - B_e)$$

So now we have

$$I_1 = G P_{in} R_d \quad I_0 = G P_{in} P_d \cdot R_{ext}$$

$$\sigma_0^2 = \sigma_{th}^2 + \sigma_{sn0}^2 + \sigma_{s-sp0}^2 + \sigma_{sp-sp}^2$$

↑
calculated w/ Power = $G P_{in} \cdot (R_{ext}) + P_{ASE}$

$$\sigma_1^2 = \sigma_{th}^2 + \sigma_{sn1}^2 + \sigma_{s-sp1}^2 + \sigma_{sp-sp}^2$$

σ_0^2 & σ_1^2 getting closer in value